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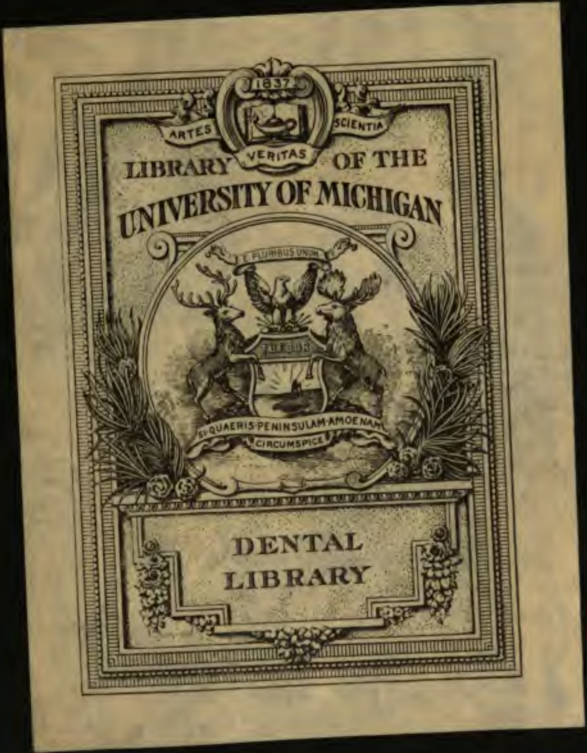

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# PREFACE.

This little book is intended for those students whose knowledge of Mathematics is limited to an acquaintance with Elementary Geometry, the solution of Simple Algebraic Equations, and a few fundamental propositions in Trigonometry. It is therefore of a strictly elementary character.

The numerous examples which illustrate the text have been, to a large extent, selected from University and College Examination Papers.

The method of treatment is identical with that pursued in the text-books already written by the Author, in which his object has been to make the subject as attractive as possible to the student.

13, Trinity College,
Dublin,
*March 22nd, 1915.*

290915

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# CHAPTER I.

## REFLECTION OF LIGHT.

1. **Light.**—Light is that agent which, by its action on the retina of the eye, is the immediate cause of our visual impressions; and **Optics** is the science which treats of the properties of this mysterious agent. Some objects such as a fire, a lighted gas-jet, the sun, or the fixed stars are *luminous*, or, in other words, they are sources of light in themselves, and affect the organs of vision without the aid of light from other bodies. Many other objects are *non-luminous*—that is, they do not emit light, and can therefore only be seen by means of rays from a luminous body which shine on them and are simply reflected from their surfaces, and, being scattered in every direction, some of them reach the eye of the observer. Thus books, flowers, and stones are non-luminous; they can only be seen by means of light from the sun, a lamp, or other luminous body; and when brought into a darkened room, they become immediately invisible. The moon, and the planets or wandering stars, are also examples of non-luminous bodies, as the light which reaches us from them is only reflected sunlight.

2. It is a matter of common experience that light passes through some bodies such as air, glass, or water; such

bodies are said to be *transparent.* Other bodies through which no light can pass are called *opaque.* The space or substance through which light passes is known as a *medium,* which may be either a vacuum or a transparent body. In the latter case, the medium is said to be *homogeneous* when its density and composition are the same in all its parts.

When air is exhausted from the glass receiver of an air-pump, objects in its interior are seen as clearly as before. Hence the presence of air or any material substance is not necessary for the transmission of light. Indeed this is evident from the fact that the light from the fixed stars reaches us across the vast depths into which they are plunged in space, there being no intervening substance which possesses the properties of matter to serve as a material medium for the transmission of these swift and mysterious messages.

It is therefore assumed that there exists, permeating all space, a subtle, imponderable, and highly elastic medium, so elastic that it is able to transmit vibrations not only of light, but also those due to other forms of energy, with enormous velocity from one part of the universe to another—a medium which occupies not only the realms of vacant space, but also pervades the interior of all bodies, even the densest solids. This medium is called the **Ether**.

The ether cannot be regarded as being at all similar to a fluid; for the vibrations which it serves to propagate are not such as are produced in fluids. It rather possesses the properties, in so far as undulatory or wave movement is concerned, of a highly elastic solid.

3. **Rectilinear Propagation of Light.**—In a homogeneous medium light is propagated in straight lines. This can be illustrated by placing two screens, each pierced with a hole on the same side of a candle, in such a position that the eye placed behind them can see the flame of the candle through the holes; this is only possible when the candle, the two holes, and the eye are in the same straight line. It is now found that when a slight lateral displacement is given to either the candle, one of the screens, or the eye, the flame of the candle disappears from view. In this case the air is the homogeneous medium, and similar experiments can be made with water or any other medium of a like nature.

## *Definitions.*

4. (i) A **Ray** is the smallest conceivable portion of light, and is, when passing through a homogeneous medium, represented graphically by the straight line along which it is propagated.

(ii) A **Pencil of Light** is a number of rays all of which proceed to or from a single point. When the rays all proceed to a point they constitute what is called a *convergent pencil*; and when they emerge from a point, the pencil is said to be *divergent*.

(iii) The point of intersection of a pencil of rays is called a **Focus**.

(iv) **Reflection of Light.**—When a ray of light *AB* (fig. 1) falls on a polished opaque surface *MN*, such as a mirror, it is turned back or reflected along the line *BC*. The ray *AB* is called the *incident ray*, and *BC* the *reflected ray*. The perpendicular *BP* to the surface at the point *B* is called the *normal* at that point.

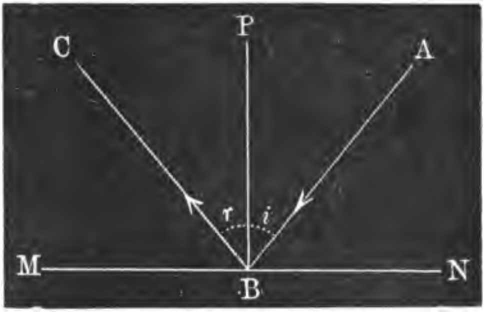

FIG. 1.

(v) The angle *i* between the incident ray and the normal is called the *angle of incidence*, and the angle *r* between the reflected ray and the normal the *angle of reflection*.

(vi) The plane *ABP* containing the incident ray and the normal is called the *plane of incidence*, and the plane *CBP* containing the reflected ray and the normal the *plane of reflection*.

*Laws of Reflection.*

5. (i) **The Planes of Incidence and Reflection coincide.**—This means that the incident ray, the normal, and the reflected ray all lie in one plane.

(ii) **The angle of incidence is equal to the angle of reflection.**

### *Experiment to Illustrate Laws of Reflection.*

Let a graduated circle be taken and immersed vertically in a basin of mercury, or any other reflecting liquid, so that a diameter *AB* of the circle coincides with the horizontal surface of the liquid, and let two tubes or telescopes *C* and *E* on the circumference slide along the arc having in all positions their axes directed along radii of the circle. If now the tube *C* be directed to a star or any bright object *S*, rays of light from *S* proceed down this tube, striking the surface of the mercury at *O*, the angle *COP* (*OP* being the normal at *O*) being the angle of incidence *i*. Let the tube *E* now be moved round until the observer's eye, looking through at *E*, sees the image

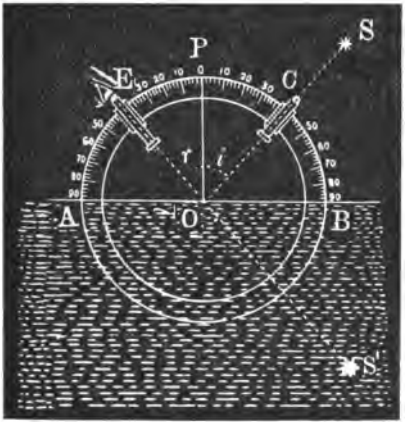

FIG. 2.

of $S$ which will be observed in the direction $EOS'$; then $OE$ represents the direction of the reflected ray, and the angle $POE$ is the angle of reflection $r$. It is now found that the arc $CP$ contains the same number of graduations as the arc $EP$—that is, the angle $i$ is equal to the angle $r$; and also $OC$, $OP$, and $OE$ evidently lie in the same plane, which is a plane parallel to that of the graduated circle itself.

6. **Reflection of a Pencil of Light from a Plane Mirror.**—Let a ray of light $AO$ fall on a plane mirror $MN$ (fig. 3), being reflected along $OC$. Drop $AN$ perpendicular to the mirror and produce it, meeting $OC$ produced in $B$.

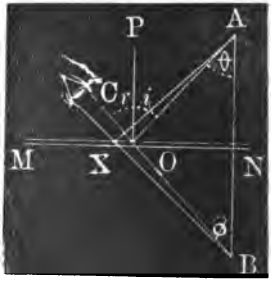

Fig. 3.

Now, by the laws of reflection (Art. 5), the $\angle i = \angle r$. But since $OP$ is parallel to $AB$, we have

$$\angle i = \angle \theta, \quad \text{and} \quad \angle r = \angle \phi;$$

hence $\angle \theta = \angle \phi$, and the angles at $N$ are both right. Also, $ON$ is common to the two triangles $AON$ and $BON$.

$$\therefore \; AN = BN.$$

Similarly, if we take any other ray $AX$, it is also reflected as if coming from the same point $B$. Hence *a pencil of rays proceeding from $A$ will, after reflection from the mirror, appear to have come from a point $B$ at a perpendicular distance behind the mirror equal to the perpendicular distance of $A$ in front of the mirror.*

7. **Conjugate Foci.**—The point *A* from which the pencil of rays springs (fig. 3) is called the *incident focus* or *radiant point*, and the point *B* the *conjugate focus*, the *image*, or the *reflection* of *A*. Taken together, the two points *A* and *B* are termed *conjugate foci*. We shall presently see that these terms not only apply to plane mirrors, but are also applicable to spherical and other mirrors, and also to lenses—in fact, to all cases in which a pencil of light proceeds from one point and is brought to a focus at the other.

When any object is placed in front of a plane mirror, each point in the surface of the body sends out a pencil of light which, after reflection from the surface of the mirror, enters the eye as if it came from the conjugate focus of the point from which the light emerged. The collection of these innumerable conjugate foci forms the *image* of the body. Hence—

**Def.**—An *image* of an object is the collection of all the conjugate foci of the points composing the surface of the object.

**Note.**—This definition, as will presently be shown, applies not only to plane mirrors, but also to other forms of mirrors and lenses.

8. **Real and Virtual Images**—We have seen in Art. 6 that a pencil of rays, proceeding from *A* and reflected from the plane mirror *MN*, enters the eye as if the rays came from *B*. The rays of the pencil, however, do not actually pass through the conjugate focus *B*. Such a focus is called a *virtual* focus; while a focus through which the rays after reflection actually pass is called a *real* focus.

### *Images in Plane Mirrors.*

9. Let *MN* be the plane mirror, and *AB* the object. Drop a perpendicular from *A* on the plane of the mirror and produce it to *A′*, an equal distance behind the mirror;

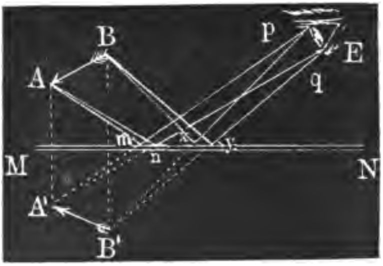

FIG. 4

then *A′* is the conjugate focus of *A*. Then if *pq* be the pupil of the eye *E* of the observer, join *A′* to *p* and *q*, cutting the mirror in *m* and *n*; then *Am* and *An* are the extreme rays of the pencil proceeding from *A*, which can, after reflection, enter the eye. Similarly, a pencil from *B*, after reflection, enters the eye as if coming from *B′*, and the same holds for each point of *AB*. Hence *A′B′* is the image of *AB*. The image appears therefore of the same size as the object, and is *virtual.*

The images in plane mirrors are *erect*—i.e. not inverted. They are, however, not exact facsimiles of the objects, as they differ in the same way as the left foot differs from the right. In fact, when the impress of a freshly written letter is taken on a blotting-pad, it can easily be deciphered on the pad by holding it up before a plane mirror, when the writing appears written in the same direction as in the original letter.

**Note.**—It is evident that only the part *mn* of the mirror is involved in producing the image of *A*, and the part *xy* in the case of *B*.

### *Images formed by two Plane Mirrors inclined to one another at any angle.*

10. Let *OM*, *ON* be perpendicular sections of the two plane mirrors, and *A* be any radiant point within the angle formed by the mirrors. With *O* as centre and *OA* as radius, describe a circle, when it will be seen that two sets of images of *A* will be formed, all lying on the circumference of this circle.

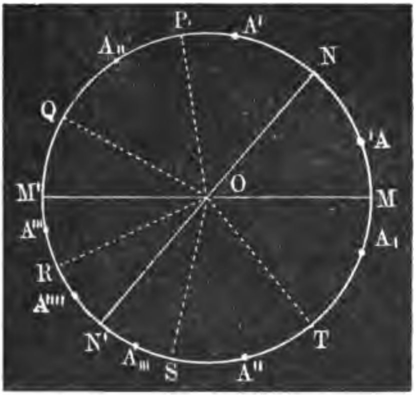

FIG. 5.

**First Set of Images.**—Now $A_{\prime}$ will be the first image of *A* formed by the mirror *OM*; $A_{\prime}$ being determined by dropping a perpendicular from *A* on *OM*, and producing out an equal distance on the opposite side; and since $A_{\prime}$ and *A* are evidently equally distant from *O*, it follows that $A_{\prime}$ will lie on the circumference of the circle. Again, an image of $A_{\prime}$ is formed at $A_{\prime\prime}$ by the mirror *ON*, $A_{\prime\prime}$ being determined by dropping a perpendicular from $A_{\prime}$ on *ON* and producing for an equal distance on the other side of $ON$; $A_{,,}$ is thus an image of an image, and also lies on the circumference of the circle. In like manner an image of $A_{,,}$ is formed at $A_{,,,}$ by the mirror $OM$ (the perpendicular being dropped from $A_{,,}$ on $OM$ produced if $A_{,,}$ is situated as in fig. 5), $A_{,,,}$ also lying on the circumference of the circle. Also $A_{,,,,}$ will be the image of $A_{,,,}$ formed by the mirror $ON$; and since $A_{,,,,}$ lies within the angle $M'ON'$ formed by producing $OM$ and $ON$, and therefore lies at the back of both mirrors, no light proceeding from it can fall on either mirror, and hence $A_{,,,,}$ is the last of this series.

**Second Set of Images.**—In the same manner $A$ has an image $A'$ formed by mirror $ON$, $A'$ has an image $A''$ formed by mirror $OM$, also $A'''$ will be the image of $A''$ formed by mirror $ON$; and if $A'''$ is found to be inside the angle $M'ON'$ this will be the last of the series.

Also, if angles $NOP$, $POQ$, $QOR$, &c., be constructed, each equal to $MON$, it will be found that only one image lies inside each of these angles excepting those angles which overlap the angle $M'ON'$, such as the angle $ROS$, in which there may be two images.

We shall now take particular cases in which the angle $MON$ is contained an exact number of times in 360°.

11. **Two Mirrors at Right Angles.**—There will be an image of $A$ at $A_1$ formed by the mirror $OM$ (fig. 6), also an image of $A_1$ at $A_2$ formed by the mirror $ON$, which is the last of the first series. Again there will be an image of $A$ at $A_3$ formed by the mirror $ON$, and also an image of $A_3$ formed by the mirror $OM$; but this image will coincide with $A_2$, and will therefore not be a new image. Hence there are *three images.*

To trace the path of the rays in each case, let $E$ be the position of the eye. Drop a perpendicular from $A$ on $OM$, and produce it out to $A_1$, an equal distance behind $OM$. Join $A_1E$ to $E$, the eye of the observer, cutting the mirror in $M$ and join $AM$. Then $AME$ is the path of one of the rays of the pencil by which the image $A_1$ is formed. Similarly $ANE$ is the path of a ray by which $A_3$ is formed by the mirror $ON$.

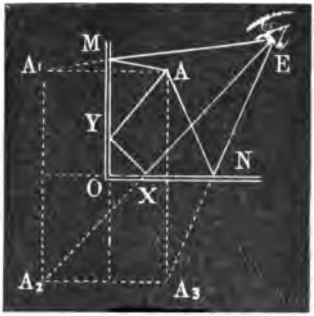

FIG. 6.

Again drop a perpendicular from $A_1$ on $ON$ produced, and produce an equal distance to $A_2$. Join $EA_2$, which will intersect $OM$ or $ON$. If, as in fig. 6, it intersects $ON$ in $X$, then $A_2$ will be the image of $A_1$. Draw $XY$ so that $XY$ produced would pass through $A_1$, and join $AY$. Then the path of a ray by which $A_2$ is seen proceeds from $A$ along $AY$: it is then reflected along $YX$, as if it came from $A_1$; and it is again reflected along $XE$ to the eye of the observer, as if it came from $A_2$.

**Angle of 60°.**—At the point $O$ (fig. 7) we can construct six angles each equal to 60°.

Hence, if an object, e.g. a letter $F$, is placed in the angle between the mirrors, there are five images seen.

12. The student will now observe that 90° is contained in 360° four times, and we have three images. Similarly, 60° is contained in 360° six times, and we have five images. Generally, if the angle between the mirrors is contained an exact number of times—say, *n* times—and if *n* be even, as it was in the last two examples, then there will be $n - 1$ images, as the last image of each series will overlap. However, if *n* be odd, the last of each series will not overlap, and there will therefore be *n* images, unless the object be placed on the bisector of the angle between the mirrors, in which case there are $n - 1$ images.

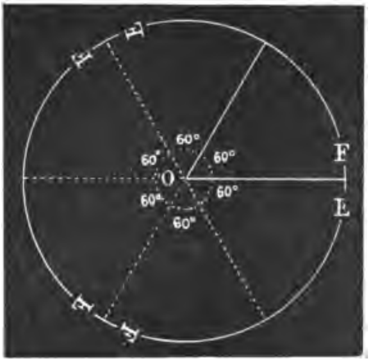

FIG. 7.

If the angle between the mirrors is contained in 360° *n* times plus a remainder, the number of images will at least be *n*; and, for certain positions of the object, the number will be $n + 1$.

The student can easily verify these rules by a construction similar to that in fig. 5, by placing the mirrors at certain angles, such as 18°, 45°, 72°, or 120°, so that *n* may be an even or an odd integer; and also at an angle such as 80°, which is not contained an exact number of times in 360°.

13. **The Kaleidoscope.**—The symmetrical arrangement of the object and its five images, when two mirrors are inclined at an angle of 60°, is the principle of the kaleidoscope. It consists of a tube inside which are fitted two plane mirrors extending along its whole length, and inclined to one another at an angle of 60°. One end of the tube is closed with a metal plate, pierced by a hole through which the observer looks; at the other end are two glass plates—one of ground, and the other of clear glass: this latter plate being next the eye. These plates are close together, with a number of small pieces of coloured glass or other bright objects lying loosely between them. Each coloured object forms five images, and all the objects, together with their images, form a symmetric pattern of great beauty, which can be made to pass through an infinite variety of changes by shaking or turning the tube so as to alter the relative positions of the objects. Sometimes a third mirror is added, inclined at an angle of 60° to each of the others, so that a cross-section of the three forms an equilateral triangle, when the number of images is greatly increased, as there are then three pairs of mirrors, each pair forming a symmetric pattern of images.

14. **Deviation of a Ray.**—When the direction of a ray of light is altered one or more times by any cause whatever, then the angle between the original and final directions is called the *deviation* of the ray.

### *Hadley's Sextant.*

15. This instrument consists of a fixed framework formed by a graduated arc *AB* (fig. 8), and two fixed arms *AM* and *BM*, *M* being situated at the centre of the circle formed by *AB*. A movable arm *VM* turns round the centre *M*, its extremity *V* travelling along the graduated arc *AB*. This movable arm carries a small mirror at *M* which moves with the arm. On one of the fixed arms *BM* is a fixed mirror *N* whose plane is parallel to the other fixed arm *AM*, so that when the movable arm *VM* coincides with *AM* the two mirrors at *M* and *N* are parallel—that is, their angle of inclination is zero. The point *A* is, for this reason, the zero of the

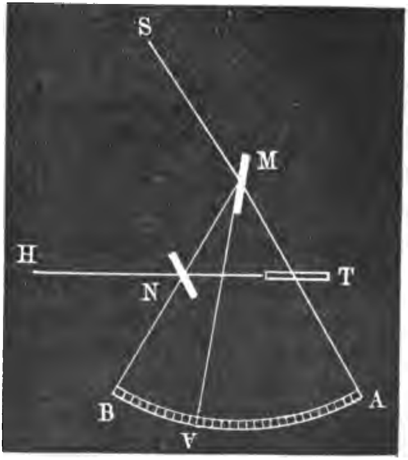

FIG. 8.

scale. Half of the mirror *N* is silvered, so as to act as a mirror; the other half is plain glass, and therefore transparent. The planes of both mirrors are perpendicular to the plane of the instrument. A small telescope *T* is fixed to the arm *AM*, and directed to the mirror *N*.

This instrument is used for measuring the angle which two objects subtend at the eye. Supposing *H* and *S* to be the two objects, the sextant is held so that its plane passes through both objects, and in such a position that, on looking through the telescope *T*, one of the objects *H* can be seen through the unsilvered half of the mirror *N*. The mirror *M* is then rotated by means of the arm *VM* until the image of *S* coincides with *H*. In this case the rays from *S* are doubly reflected, first from the mirror *M*, and then from the silvered half of *N* along the lines *SMNT*. Having got the image of *S* coincident with *H*, the arm *VM* is clamped, and the arc *AV* is read off by means of a vernier at *V*. This reading, on being doubled, gives the angular distance between the objects. Usually, however, the half degrees are numbered as whole ones, so that the reading on the instrument at once gives the required angle.

The arc *AV* evidently measures the angle at which the mirrors are inclined to one another, for it measures the angle between *AM* and the mirror at *M*, *AM* being parallel to the mirror at *N*.

16. The principle on which Hadley's Sextant depends is that, *when a ray has been doubly reflected from two mirrors in succession, the deviation of the ray is double the angle between the reflecting surfaces of the mirrors.*

To prove this, let *SMNO* be the path of the ray during its double reflection from the mirrors *M* and *N*. Now *SM* and *MN* make equal angles with the mirror *M*; let each of these angles be $a$.

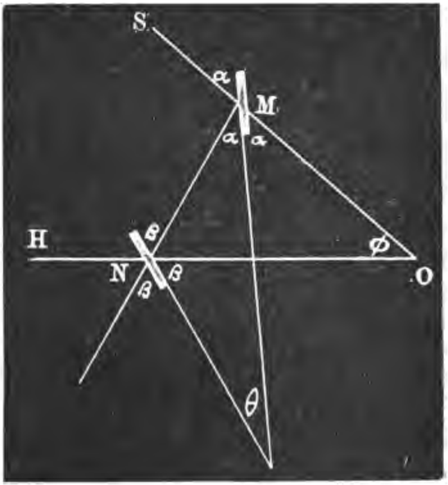

FIG. 9.

Also let *MN* and *NO* each make an angle $\beta$ with the mirror *N*.

Let the original and final direction of the rays meet at *O*; then the angle $\phi$ is the deviation of the ray, and $\theta$ is the angle between the mirrors. We have to prove $\phi = 2\theta$. Now we have (*Euclid*, I, 32).

The external angle $2\beta = 2a + \phi$;

$$\therefore \quad \phi = 2\beta - 2a = 2(\beta - a).$$

For the same reason

$$\beta = a + \theta;$$

$$\therefore \quad \theta = \beta - a;$$

$$\therefore \quad \phi = 2\theta.$$

## EXERCISES I.

1. Two parallel plane mirrors face each other at a distance of 3′, and a small object is placed between them at a distance of 1′ from the first mirror. Calculate the distances from the first mirror of the three nearest images which are seen in it, and make a similar calculation for the second mirror.

2. A small mirror hangs on a wall: show that a man standing opposite to the mirror will just be able to see the image of his feet if the height of the mirror above the ground is half the height of his eye above the ground.

3. What is the smallest size of a plane mirror in which a person can just see his whole body?

4. A square plane mirror hangs with its centre coinciding with the centre of one of the walls of a cubical room. What must be the size of the mirror in order that an observer with his eye exactly in the centre of the room may *just* be able to see the whole of the opposite wall, except the part concealed by his body?

5. A man whose eye is 5 feet 6 inches above the ground sees the reflection of the top of a mountain in a small pool of water on the pathway. The pathway is level; and he finds that the pool is 11 yards distant, while the map shows the mountain to be 6 miles away. Find the height of the mountain.

6. If two plane mirrors are inclined to one another at an angle of 18°, how many images are seen of a small object placed between them?

7. If two plane mirrors are inclined to one another at an angle of 72°, how many images are seen of a small object placed between them (1) when the object is placed on the line bisecting the angle between the mirrors, (2) when the object is placed in any other position?

8. Two plane mirrors contain an angle of 160°. Show that, if the object be placed within 20° of either mirror, there will be three images, and that if it be more than 20° from both mirrors, there will be only two.

9. A ray of light is reflected in succession from two plane mirrors inclined to one another at a fixed angle. Show that the deviation of the ray—i.e. the angle between its original and final directions—is twice the angle between the mirrors.

10. Find the angle between two plane mirrors in order that, after reflection from each of them in succession, the final direction may be (1) at right angles to its original direction; (2) inclined to the original direction at an angle of 120°.

## *Spherical Mirrors.*

17. A *spherical mirror* is one whose reflecting surface is a portion of the surface of a sphere. It is *concave* or *convex* according as the inside or outside of its surface is made use of to reflect the light. The centre $C$ of the sphere, of which the mirror is a portion (figs. 10, 11), is called the *centre of curvature* of the mirror.

18. **Reflection of a Ray of Light from a Spherical Mirror.**—Let $AO$ represent a ray of light proceeding from a luminous point $A$ (fig. 10), and falling on a *concave* mirror whose centre of curvature is $C$. It is reflected at $O$ along $OB$ intersecting $AM$, the line joining $A$ to the centre of curvature, in $B$. And, since the radius $CO$ is perpendicular to the tangent at $O$, $CO$ is therefore the normal to the mirror at $O$, the angles $i$ and $r$ being the angles of incidence and reflection respectively; and these two angles are equal to one another (Art. 5). Now, since $OC$ bisects the

vertical angle of the triangle *AOB*, it divides the base *AB* in the same ratio as the sides (*Euclid* VI, 3); hence

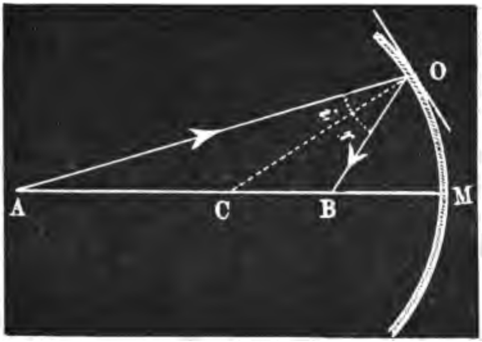

FIG. 10.

$$AO : BO :: AC : BC.$$

It is to be borne in mind that all rays proceeding from *A* do not, after reflection, proceed to the same point *B*, as the position of the point *B* depends on the ratio *AO* : *BO*, which is different for different positions of the point *O*; but for those rays of light from *A* which fall almost perpendicularly on the mirror, or, in other words, when the point *O* is very near the point *M*, then the above proposition becomes

$$AM : BM :: AC : BC,$$

that is, *when O is close to M the distances of A and B from the mirror are in the same ratio as their distances from the centre of the mirror.*

The same law holds also for a convex mirror (fig. 11), the incident ray *AO* being reflected along *OK* (making $\angle r = \angle i$), which, when produced backwards, meets *AC* in *B*. In this case the radius *CO* produced bisects the external vertical angle of the triangle $AOB$; hence it divides $AB$ externally in $C$ in the same ratio as the sides; and we have as before

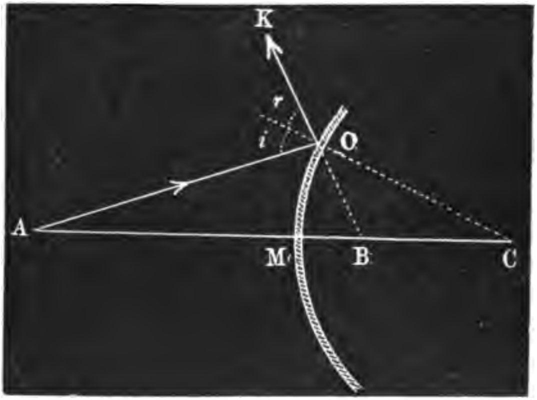

FIG. 11.

$$AO : BO :: AC : BC,$$

or, when $O$ is close to $M$,

$$AM : BM :: AC : BC.$$

19. **Conjugate Foci.**—We have seen that when a pencil of rays proceeds from a fixed point $A$ (figs. 10, 11) in front of a spherical mirror, and the rays fall perpendicularly on the surface of the mirror, they are either reflected to a fixed point $B$ (fig. 10), or as if they had come from a fixed point $B$ (fig. 11). The relation between these two points is evidently mutual; for, if in fig. 10 the ray proceeded from $B$ along $BO$, it would be reflected back to $A$; and in fig. 11, if the path of the ray had been along $KO$, in the direction of $B$, it would be reflected back to $A$.* On account of this mutual relation $A$ and $B$ are called *conjugate foci.*

---

* This illustrates an important principle in Optics, that, if a ray of light be reversed in direction, it will describe the same path backwards as it traversed before being reversed.

20. **Principal Focus.**—If in Art. 18 the incident rays be parallel to the radius *CM*, and if the rays fall almost perpendicularly on the mirror, or, in other words, if the point *O* for each ray be very close to *M*, then the rays, after reflection, will cut the radius *CM* approximately in the same point *F*. This point is called the principal focus corresponding to the radius *CM*. Hence the following definition:—

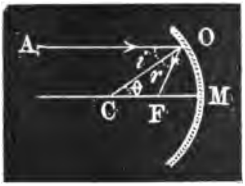

FIG. 12.

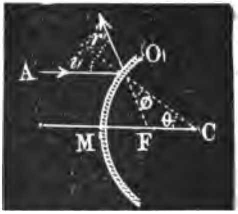

FIG. 13.

**Def.**—If rays parallel to a radius of a mirror fall almost perpendicularly on the surface of the mirror, the point in which the rays, after reflection, cut the radius is called the *Principal Focus* of the mirror corresponding to that radius.

*To show that the Principal Focus of a Spherical Mirror corresponding to any Radius is the middle point of that Radius.*

21. For we have, for a *concave* mirror (fig. 12), the $\angle i = \angle r$; but $\angle i = \angle \theta$, $\therefore \angle \theta = \angle r$; $\therefore CF = FO$. But since *O* is close to *M*, *FO* and *FM* nearly coincide.

Hence, approximately, $CF = FM$.

In the case of a convex mirror (fig. 13), the $\angle i = \angle r$; but $\angle i = \angle \theta$, and $\angle r = \angle \phi$; $\therefore \angle \theta = \angle \phi$; $\therefore CF = FO$, i.e. $CF = FM$, as before.

**Def.**—The *focal length* of a mirror is the distance of any principal focus from the mirror measured along the radius to which it corresponds.

The focal length of a mirror is therefore equal to half the radius; and if we regard lines measured from the mirror towards the incident light as positive, and from the incident light as negative, the focal length is evidently *positive* for *concave*, and *negative* for *convex* mirrors. Hence if $f$ denote the focal length and $r$ the radius of a mirror, we have

$$f = \pm \frac{r}{2},$$

the plus or minus sign being taken according as the mirror is concave or convex respectively.

22. **To prove the Equation**

$$\frac{1}{d} + \frac{1}{D} = \frac{2}{r},$$

where

$D$ = distance of the incident focus $A$ from the mirror,
$d$ = distance of its conjugate focus $B$ from the mirror, and
$r$ = radius of the mirror.

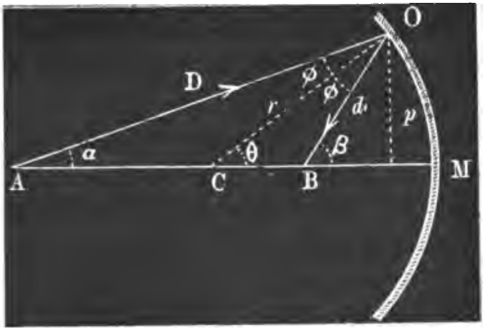

FIG. 14.

**Case I. Concave Mirror.**—Let $AM$ (fig. 14) be the distance of the incident focus $A$ from the mirror $= D$; and when $O$ is very close to $M$, then $AO = AM = D$. Similarly, $BO = BM = d$. Let the angles of incidence and reflection each $= \phi$, and the angles $\alpha$, $\beta$, and $\theta$ as marked on the diagram. Then $\alpha + \phi = \theta$.

$$\therefore \quad \alpha = \theta - \phi;$$

but $$\beta = \theta + \phi.$$

Add $$\therefore \quad \alpha + \beta = 2\theta.$$

But $\alpha$, $\beta$, and $\theta$ being small, therefore these angles, expressed in circular measure, are approximately equal to their sines. Hence, if $p$ be the perpendicular dropped from $O$ on $CM$, we have

$$\alpha = \frac{p}{D}, \quad \beta = \frac{p}{d}, \quad \text{and} \quad \theta = \frac{p}{r};$$

$$\therefore \quad \frac{p}{D} + \frac{p}{d} = \frac{2p}{r}, \quad \text{or} \quad \frac{1}{d} + \frac{1}{D} = \frac{2}{r}.$$

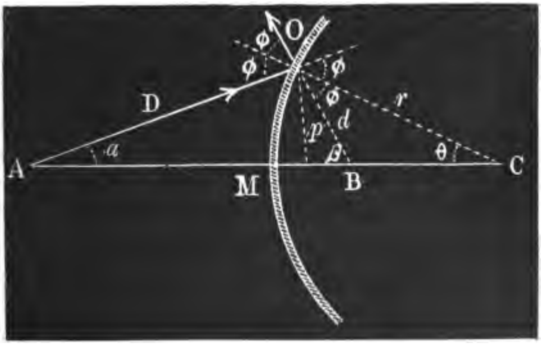

FIG. 15.

**Note.**—It is left as an exercise for the student to prove the truth of the above equation for a concave mirror where the incident focus $A$ is between the principal focus and the mirror.

**Case II. Convex Mirror.**—Here we have (fig. 15)

$$\alpha + \theta = \phi, \qquad \therefore \quad \alpha = \phi - \theta;$$

but $\quad \beta = \phi + \theta.$

Subtract $\quad \therefore \quad \alpha - \beta = -2\theta.$

In this case both $d$ and $r$ must be regarded as negative, being measured from the incident light;

$$\therefore \quad \alpha = \frac{p}{D}, \quad \beta = \frac{p}{-d}, \quad \text{and} \quad \theta = \frac{p}{-r};$$

$$\therefore \quad \frac{p}{D} - \frac{p}{-d} = -\frac{2p}{-r},$$

or, $$\frac{1}{d} + \frac{1}{D} = \frac{2}{r}, \quad \text{as before.}$$

23. We thus see that the equation holds whether the conjugate focus $B$ be real or virtual, due regard being had to the signs of $D$, $d$, and $r$, which are counted *positive* if measured *from* the mirror in the *opposite* direction to that in which the incident light is moving, and *negative* if in the *same* direction as that of the incident light. This means that any distance measured in front of the mirror is positive, and behind the mirror negative.

It is to be borne in mind in working the following examples that, when it is said that the light *converges* to a point a certain distance *behind* the mirror, the incident focus in this case is virtual, for it is as if we imagine a convergent pencil of light proceeding in direction $KO$, each ray being incident near $O$ (fig. 11) such that if the mirror were removed they would meet in $B$, which is the virtual incident focus; they are then reflected at $O$ and meet at $A$, the conjugate focus. In this case the distance of the given virtual focus from the mirror is taken as negative.

Since $r = 2f$, where $f$ is the focal length (Art. 21), the above equation can be written thus:—

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f}.$$

Ex. 1. A pencil of light *diverges* from a point 6″ in front of a *concave* mirror whose radius is 4″: find the position of the conjugate focus.

Since the light diverges from a point 6″ in front of the mirror, $D = 6''$, and is positive; and, since the mirror is concave, $r$ is positive. Hence we have

$$\frac{1}{d} + \frac{1}{6} = \frac{2}{4} = \frac{1}{2},$$

$$\therefore \quad \frac{1}{d} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3};$$

$$\therefore \quad d = 3''.$$

Hence, the conjugate focus is 3″ from the mirror; and, since the answer is positive, it is situated in front of the mirror.

Ex. 2. Light *diverges* from a point 6″ in front of a *convex* mirror of radius 4″: find the position of the conjugate focus.

Here $D = 6''$, and $r = -4''$ (the mirror being convex);

$$\frac{1}{d} + \frac{1}{D} = \frac{2}{r},$$

or,

$$\frac{1}{d} + \frac{1}{6} = -\frac{2}{4} = -\frac{1}{2};$$

$$\therefore \quad \frac{1}{d} = -\frac{1}{2} - \frac{1}{6} = -\frac{2}{3};$$

$$\therefore \quad d = -\frac{3}{2} = -1\tfrac{1}{2} \text{ inches}.$$

Hence, the conjugate focus is $1\frac{1}{2}''$ from the mirror; and, since the answer is negative, it is situated behind the mirror.

Ex. 3. If the radius of a *concave* mirror be 4″, and if the incident pencil be a number of rays *converging* to a point 6″ from its surface, find the position of the conjugate focus.

Here, since the light converges, $D = -6''$, and $r = 4''$; and

$$\frac{1}{d} - \frac{1}{6} = \frac{2}{4} = \frac{1}{2},$$

$$\therefore \quad \frac{1}{d} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3};$$

$$\therefore \quad d = \frac{3}{2} = 1\tfrac{1}{2}'' \text{ in front of the mirror.}$$

Ex. 4. A pencil of light falls on a convex mirror of radius 4″, and the direction of the rays is such that, if the mirror were removed, the rays would *converge* to a point 6″ from the surface of the mirror: find the position of the conjugate focus.

Here $$D = -6'', \quad \text{and} \quad r = -4'',$$

and $$\frac{1}{d} - \frac{1}{6} = -\frac{2}{4} = -\frac{1}{2};$$

$$\therefore \quad \frac{1}{d} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3};$$

$$\therefore \quad d = -3'' \text{ behind the mirror.}$$

Ex. 5. A pencil of light *diverging* from a point 10″ in front of a mirror appears to *diverge*, after reflection, from a point situated 3″ *behind* the mirror: find the radius of the mirror, the focal length, and the nature of its curvature.

Here, $$D = 10'', \quad \text{and} \quad d = -3'',$$

and $$-\frac{1}{3} + \frac{1}{10} = \frac{2}{r};$$

or, $$-\frac{7}{30} = \frac{2}{r}, \quad \text{or} \quad -7r = 60.$$

$$\therefore \quad r = -8{\cdot}57''.$$

Also, the focal length $f = \frac{r}{2} = -4{\cdot}285''$; and the curvature is convex.

### EXERCISES II.

1. A pencil of light is incident on the surface of a *convex* mirror of 6″ radius, and the direction of each ray is such that, if the mirror were removed, they would all *converge* to a point 10″ from the mirror: find the position of the conjugate focus.

2. A pencil of light is incident on a *concave* mirror of radius 10″, and the direction of each ray is such that, if the mirror were removed, they would meet in a point 14″ from the mirror: find the position of the conjugate focus.

3. Light *diverges* from a point 6″ from a *convex* mirror of radius 10″: find the position of the conjugate focus.

4. Light *diverges* from a point 4″ distant from a *concave* mirror of radius 10″: find the position of the conjugate focus.

5. What is the focal length (1) of a concave mirror, (2) of a convex mirror, the radius of the mirror in each case being 1 foot?

6. Light *diverging* from a point 20″ from a mirror is reflected to a point 6″ in front of the mirror: find the radius and curvature of the mirror. Also, what is its focal length?

7. Light *diverging* from a point 20″ from a mirror is reflected in such a manner that the rays seem to come from a point 6″ behind the mirror: find the radius and curvature of the mirror. Also, what is its focal length?

24. The relative positions of the two conjugate foci can be determined by the equation in Art. 22, and from it we could deduce the changes in the position of the conjugate focus $B$ as the incident focus $A$ varies its position. These changes, however, can be more easily seen by the aid of the following equation:—

Proof of the equation $\delta\delta' = f^2$, where

$\delta$ = the distance of $A$ from the principal focus $F$,

$\delta'$ = the distance of $B$ from the principal focus $F$,

and $f$ = the focal length of the mirror [figs. 16(a), 16(b)].

**Case I. Concave Mirror** [fig. 16 (a)].—Here $AF$ and $BF$ are $\delta$ and $\delta'$ respectively; and since

$$FC = FM = f, \text{ hence } AM = \delta + f, \text{ and } BM = f + \delta'.$$

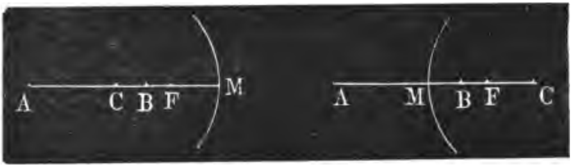

Fig. 16 (a). Fig. 16 (b).

Also, $AC = \delta - f$, and $BC = f - \delta'$.

Now, since $AM : BM :: AC : BC$ (Art. 18).

Hence $\delta + f : f + \delta' :: \delta - f : f - \delta'$;

therefore, multiplying the extremes and means, we have

$$(\delta + f)(f - \delta') = (\delta - f)(f + \delta'),$$

and eventually $\delta\delta' = f^2$.

**Note.**—It is left as an exercise for the student to prove the truth of the above equation when the incident focus $A$ is between the principal focus and the mirror.

**Case II. Convex Mirror** [fig. 16(b)].—

Here $AM = \delta - f$, $BM = f - \delta'$, $AC = \delta + f$,

and $BC = f + \delta'$.

Hence $\delta - f : f - \delta' :: \delta + f : f + \delta'$;

and, multiplying the extremes and means, we get, as before,

$$\delta\delta' = f^2;$$

that is, $AF . BF = FC^2 = FM^2$.

25. From this equation the following conclusions can easily be deduced :—

(1) Since $f^2$ is always positive, even when $f$ itself is negative, therefore $\delta\delta'$ is always positive; hence $\delta$ and $\delta'$ must both have the same sign, either both positive or both negative. Therefore, *both conjugate foci A and B must, in all cases, lie on the same side of F.*

(2) Since $\delta\delta' = f^2$, it follows that, if $\delta$ increases, $\delta'$ decreases, and *vice versa.* Hence *if one of the two conjugate foci A and B approach the principal focus F, the other must recede from it.*

(3) If either $\delta$ or $\delta'$ be less than $f$, the other must be greater than $f$. Hence *if either of the conjugate foci A or B lie between C and M, that is, internal to the radius, the other must lie outside CM, that is, on CM produced.*

(4) If either $\delta$ or $\delta' = O$, the other must equal infinity, for $\frac{f^2}{O}$ = infinity. Hence, if either of the conjugate foci *A* or *B* coincides with *F*, the other must be at infinity, and *vice versa.* This result is identical with the definition given of the "Principal Focus" (Art. 20).

(5) If either $\delta$ or $\delta' = f$, the other must also $= f$. Hence *the conjugate focus of the centre C coincides with C, and also that of the point M coincides with M.*

## *Images in Spherical Mirrors.*

26. **Concave Mirror. Case I.**—Where the object lies between the centre *C* and the principal focus *F*.

Let *AB* be a small object placed between *C* and the principal focus *F* of the concave mirror corresponding to the radius *CM*, each point in the object *AB* being close to *CM* (fig. 17). Through *A* draw the path of a ray *AO* parallel to *CM*; this ray is reflected along *OF* (Art. 20). Hence the conjugate focus of *A* lies on *OF* (or *OF* produced). Also, the conjugate focus of *A* lies on *AC* (or *AC* produced) (Art. 18). Hence it must be at *A′*, where *OF* and *AC* intersect. Similarly, the conjugate focus of *B* is at *B′*. And also the conjugate focus of any point between *A* and *B* can be shown to lie between *A′* and *B′*. Therefore the image of *AB* is at *A′B′*, and the eye of the observer placed behind *A′B′* will perceive an *inverted* image of *AB* at *A′B′*; and, since the rays after reflection actually pass through *A′*, *B′*, and all the intermediate conjugate foci, this image is *real*.

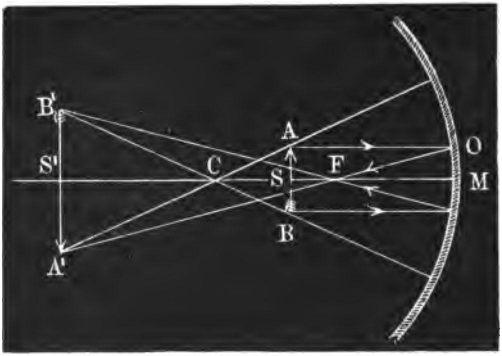

FIG. 17.

**Case II.**—When the object is outside the centre of curvature *C*. This is only the converse of the last case; and, by a similar construction, it can be shown that when the object is at *A′B′* an *inverted real* image is found at *AB*.

In fig. 18 is shown a candle-flame situated between the centre and the principal focus of a concave mirror which throws an inverted real image on a screen situated outside the centre of curvature. It is only when a real image is formed that it can be cast on a screen, in which case the observer can view the image independent of the position of his eye, provided the front of the screen is within his field of view.

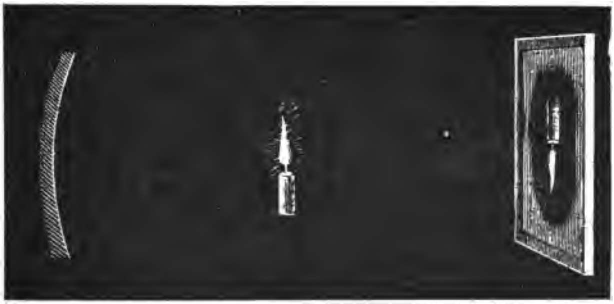

FIG. 18.

**Case III.**—Where the object lies between the principal focus *F* and the mirror.

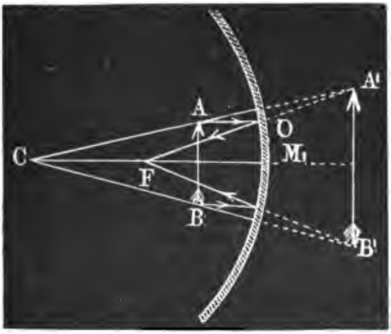

FIG 19.

As before, *AB* is the object lying between *F* and *M*. The ray *AO* parallel to *CM* is reflected to *F*. Hence the conjugate focus of *A* lies on *OF* (or *OF* produced). But it also lies on the radius drawn through *A*, i.e. on *AC* (or *AC* produced). Hence it is at *A′* where *OF* and *AC* intersect, and evidently this focus is *virtual*.

Similarly, the conjugate focus of *B* is at *B′*; and any intermediate point between *A* and *B* has a corresponding focus intermediate between *A′* and *B′*. Hence there is an *enlarged virtual* image of *AB* behind the mirror at *A′B′*.

**Note.**—We have assumed, in Case I, that *AC* and *OF* intersect at *A′* in front of the mirror; and, in Case III, behind the mirror. The reason of this is, that in the former case *AO* is greater than the focal length *CF*; but if the extremities of two parallel lines be joined, the joining lines will intersect on the same side as the smaller of the two; hence these lines intersect in front of the mirror. And, in the latter case, *CF* is greater than *AO*, and the point *A′* is therefore behind the mirror.

27. **Convex Mirror.**—In this case, it will be shown that the images of objects are in all cases behind the mirror—smaller in size than the object, erect (not inverted), and *virtual*.

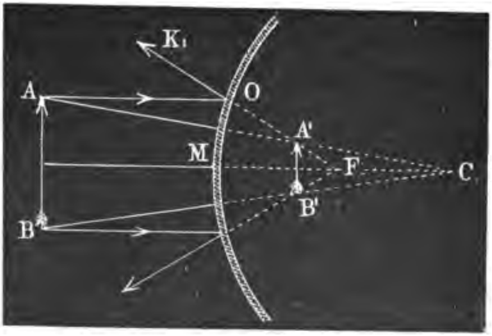

FIG. 20.

Here (fig. 20) the ray $AO$ parallel to $CM$ is reflected along $OK$ *as if it came from* $F$, and the conjugate focus must therefore lie on $OF$ (or $OF$ produced); but it also lies on $AC$. Hence the focus is at $A'$ where they intersect. Similarly, the focus of $B$ is at $B'$, and we have an *erect virtual* image at $A'B'$. In this case the image could not be thrown on a screen, which is a rule which holds for all virtual images.

28. **Magnitude of Images.**—We shall now compare any linear dimension of the object with the corresponding linear dimension of the image. To fix our ideas, we shall take a particular linear dimension, viz., the height of the object.

Let the height of object $AB$ (fig. 17) $= h$,
and let the height of image $A'B'$ $= h'$.
Then, since the two triangles $ACB$ and $A'CB'$ are similar to one another,

$$h : h' = AC : A'C = CS : CS'.$$

Hence their heights are proportional to their distances from the centre, that is (Art. 18) to their distances $D$ and $d$ from the mirror. Hence,

$$\frac{h}{h'} = \frac{D}{d},$$

$$\text{or,}\quad \frac{\text{height of object}}{\text{height of image}} = \frac{\text{distance of object from mirror}}{\text{distance of image from mirror}}.$$

Ex. 1. A candle-flame 2″ high is placed 6″ in front of a convex mirror of radius 8″: find the position and height of the image.

$$\text{Here,}\qquad \frac{1}{d} + \frac{1}{6} = -\frac{2}{8} = -\frac{1}{4},$$

$$\therefore\ \frac{1}{d} = -\frac{1}{4} - \frac{1}{6} = -\frac{5}{12}; \qquad \therefore\ d = -\frac{12}{5} = -2''{\cdot}4.$$

Hence the image is situated $2''\cdot 4$ behind the mirror.

Again,
$$\frac{h}{h'} = \frac{D}{d}, \quad \text{or} \quad \frac{2}{h'} = \frac{6}{2\cdot 4};$$
$$\therefore \quad 6h' = 4\cdot 8, \quad \text{or} \quad h' = \cdot 8''.$$

Hence the height of the image is $\cdot 8''$.

Ex. 2. Where must an object be placed in order that the height of its image formed by a concave mirror of radius $6''$ may be half the height of the object?

Here,
$$\frac{1}{d} + \frac{1}{D} = \frac{2}{6} = \frac{1}{3}.$$

Also
$$\frac{h}{h'} = \frac{D}{d}, \quad \text{or} \quad \frac{2}{1} = \frac{D}{d}; \quad \therefore \quad 2d = D, \quad \text{or} \quad d = \frac{D}{2}.$$

Hence, by substitution, we have
$$\frac{2}{D} + \frac{1}{D} = \frac{1}{3}, \quad \text{or} \quad \frac{3}{D} = \frac{1}{3};$$

therefore $D = 9''$ in front of mirror.

## EXERCISES III.

1. Show that, in the case of a convex mirror, the conjugate focus of any radiant point is behind the mirror.

2. In the case of a concave mirror, if the radiant point be situated between the principal focus and the mirror, show that its conjugate focus will be virtual and behind the mirror.

3. Show that, in the case of a concave mirror, if the distance of the radiant point from the mirror be greater than the focal length, the conjugate focus is always real and in front of the mirror.

4. Show that images in convex mirrors are always virtual and behind the mirror.

5. In what cases are the images in concave mirrors (1) real, (2) virtual?

6. A piece of wire one inch long is held at a distance of $18''$ from a concave mirror of radius $24''$: find position and magnitude of the image. Also state whether it is real or virtual.

7. A bright ball of diameter 3″ is suspended at a distance of 1 foot from a convex mirror of radius 8″: find the position, magnitude, and nature of the image.

8. A candle-flame, $1\frac{1}{2}$ inches high, is placed at a distance of 16 inches from a screen: where must a concave mirror of radius equal to 12 inches be placed in order to throw an image of the flame on the screen? What will be the size of the image, and will it be erect or inverted?

9. A grain of sand one-thousandth of an inch in diameter is placed at a distance of 5·001 inches from a concave mirror whose radius is 10 inches: find the position and magnitude of the image.

10. Find the position of the image of a small body formed by a concave mirror in the following cases:—

(1) When body is outside the centre.
(2) When body is at the centre.
(3) When body is between the centre and principal focus.
(4) When body is at the principal focus.
(5) When body is between the principal focus and the mirror.

State in each case whether the image is real or virtual.

11. Why is the image of an object formed by a plane mirror more perfect than that formed by a spherical mirror?

29. **Def.**—A *parabola* is a curve such that the distance *OF* of any point *O* on the circumference from a fixed point *F*, called the focus, is equal to its perpendicular distance *OK* from a fixed line *LK*, called the directrix. The *axis* of the parabola is the line *FL* through the focus perpendicular to the directrix.

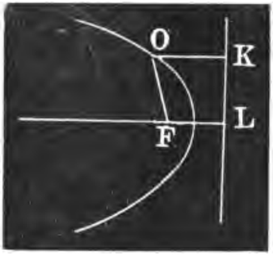

FIG. 21.

**Parabolic Mirrors.**—A *parabolic mirror* is one whose surface is generated by the revolution of an arc of a parabola *KM* about its axis *AM* (fig. 22).

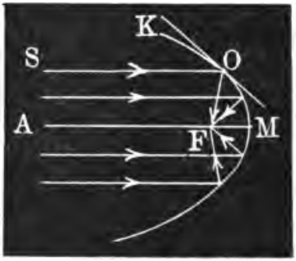

Fig. 22.

We have seen (Art. 20) that, in the case of spherical mirrors, parallel rays are only reflected approximately to the principal focus; and, conversely, when the radiant point is at the principal focus, the rays which diverge from it will, after reflection, be nearly, but not quite, parallel to one another. This is a defect from which parabolic mirrors are free. It is a property of the parabola that the straight line *OF* joining any point *O* on the curve to the focus *F*, and the line *OS* drawn through *O* parallel to the axis, make equal angles with the tangent, and therefore with the normal at *O*. Hence *all* rays parallel to the axis will, after reflection, meet at *F*; and, conversely, when a bright light of small dimensions is placed at *F*, the rays falling on the mirror will be reflected, each exactly parallel to the axis. Hence this reflected light will retain its intensity even at a great distance, for, it is the gradual divergence of these rays when reflected from a spherical mirror that is the principal cause of the loss of intensity.

For this reason, parabolic mirrors are used in lamps attached to motor-cars and railway-trains, the actual source of light being at the focus *F*. Parabolic mirrors were formerly used extensively for lighthouses, but these have been superseded by lenticular lenses, into the details of the construction of which it is not our design to enter.

# CHAPTER II.

## REFRACTION OF LIGHT.

30. **Refraction.**—When a ray of light passes obliquely from one transparent medium to another, as from air into water, it is changed in direction, so that the direction of its course in the second medium makes an angle with its course in the first. This deflection of the ray is called *refraction*. Thus, if $AO$ be a ray passing at $O$, let us say from air into water, it is bent along the direction $OB$.

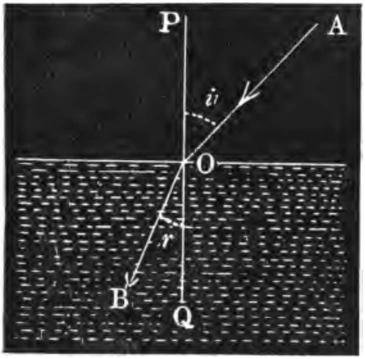

**Fig. 23.**

(i) The angle $i$ between the incident ray and the normal $OP$ is called the *angle of incidence.*

(ii) The angle $r$ between the refracted ray and the normal is called the *angle of refraction.*

(iii) The plane $AOP$ containing the incident ray and the normal is the *plane of incidence.*

(iv) The plane *BOQ* containing the refracted ray and the normal is *the plane of refraction.*

It is to be borne in mind that all the light which falls on a refracting surface does not pass through, as a portion of it is reflected from the surface. The present chapter deals exclusively with that portion of the light which passes through.

### *Laws of Refraction.*

31. (i) **The planes of incidence and refraction coincide.**—This means that the incident ray, the normal, and the refracted ray all lie in one plane.

(ii) **The sines of the angles of incidence and refraction are in a constant ratio.**

### *Experimental Demonstration of Laws of Refraction.*

Let a graduated circle be taken similar to that used in Art. 5, and immersed vertically in water or any other

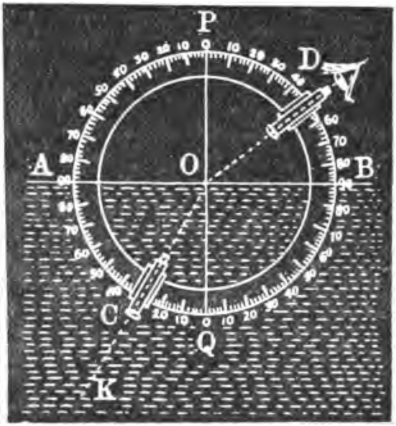

FIG. 24.

transparent liquid, so that a diameter $AB$ of the circle (fig. 24) coincides with the surface of the liquid; and let two tubes or telescopes $C$ and $D$ on the circumference slide along the arc, having in all positions their axes directed along radii of the circle. If now the tube $C$ be directed to any bright object $K$ below the surface of the liquid, and the tube $D$ be moved round until rays from $K$, passing along the radius $CO$, are refracted through the tube $D$ along the radius $OD$ to the eye of the observer, it is found, when the circle is removed from the liquid and the angles $DOP$ and $COQ$ measured, that the ratio

$$\sin DOP : \sin COQ$$

has a certain value which in the case of air and water is 4 : 3. On repeating the experiment, and placing the tube $OC$ in any other position, we find that the ratio of the sines of the angles has the same value as before. Also, it is evident that $OC$, $OD$, and the normal $PQ$ lie in the same plane, which is a plane coincident with that of the circle.

The laws of refraction as given above are commonly known as Descartes' laws. They were, however, first discovered by Snell in 1620.

32. **Relative Index of Refraction. Def.**—The constant ratio of the sine of the angle of incidence to the sine of the angle of refraction, when a ray passes from one medium into another, is called the *relative index of refraction* from the former medium to the latter.

The index of refraction corresponding to two media is generally denoted by the symbol $\mu$. Hence we have (fig. 23)

$$\frac{\sin i}{\sin r} = \mu.$$

The value of $\mu$ when light passes from air to water is $\frac{4}{3}$, and from air to glass is $\frac{3}{2}$. If, however, the light be reversed in direction, that is, if the ray *BO* be the incident ray (fig. 23), then the path of the ray is the same as before, only reversed in direction, and it is refracted along *OA*. The value of $\mu$ in this case is the reciprocal of the former value, viz. $\frac{1}{\mu}\cdot$ Thus, from water to air $\mu = \frac{3}{4}$, and from glass to air $\mu = \frac{2}{3}$. In all cases when light passes from a rarer into a denser medium, the value of $\mu$ is greater than unity, and each ray is then bent towards the normal *OQ* (fig. 23). When light passes from a denser to a rarer medium, $\mu$ is less than unity, and each ray is bent from the normal *OP*.

33. **Absolute Index of Refraction.**—When a ray of light passes from a vacuum into any medium, then the ratio of $\sin i$ to $\sin r$ is called the *absolute index of refraction* for that medium. Generally, the relative index of refraction from air into any given medium must be multiplied by the absolute index of refraction of air in order to obtain the absolute index of refraction for that medium. The absolute index of refraction of air at 0° C. and 760 mm. pressure is 1·000294.

**Note.**—In the following pages, when the index of refraction of a substance is given, it is to be understood to mean, unless the contrary be stated, the relative refractive index when the ray passes from air into the given substance.

Ex. 1. If a ray of light pass from air into a substance making an angle with the normal of 60°, and the angle of refraction be 45°, find the refractive index of the substance.

Here $$\frac{\sin i}{\sin r} = \mu, \quad \text{or} \quad \frac{\sin 60^\circ}{\sin 45^\circ} = \mu;$$

or, $$\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{1} = \mu. \quad \text{Hence} \quad \mu = \frac{\sqrt{6}}{2} = 1{\cdot}225.$$

Ex. 2. Find the angle of refraction of a ray of light incident at an angle of 60° with the normal upon a surface of fluor-spar, the refractive index of fluor-spar being 1·434.

$$\text{Here} \qquad \frac{\sin 60^\circ}{\sin r} = 1{\cdot}434, \quad \text{or} \quad \frac{\sqrt{3}}{2 \sin r} = 1{\cdot}434.$$

$$\text{Hence} \qquad 2{\cdot}868 \sin r = \sqrt{3} = 1{\cdot}732\,;$$

$$\therefore \quad \sin r = \frac{1{\cdot}732}{2{\cdot}868} = {\cdot}604.$$

On referring to a book of tables, we find $r = 37^\circ$.

Ex. 3. Find the sine of the angle of refraction of a ray of light incident at an angle of 45° with the normal on the surface of water, the refractive index of water being $\frac{4}{3}$.

$$\text{Here} \qquad \frac{\sin 45^\circ}{\sin r} = \frac{4}{3}, \quad \text{or} \quad \frac{1}{\sqrt{2} \sin r} = \frac{4}{3}\,;$$

$$\text{or,} \qquad 4\sqrt{2} \sin r = 3\,;$$

$$\therefore \quad \sin r = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8} = {\cdot}53.$$

## *Passage of a Ray through a Transparent Plate with Parallel Faces, e.g. a Sheet of Glass.*

34. Let *MN* be a plate made of glass, or other transparent substance (fig. 25), with parallel faces, and let *ABCD* be the path of the ray, being bent towards the normal on entering at *B*, and from the normal on emerging from the plate at *C*. Now, if $\mu$ be the index of refraction from air to glass, the index from glass to air is $\frac{1}{\mu}$ (Art. 32).

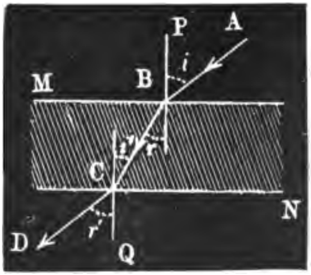

FIG. 25.

Hence

$$\frac{\sin i}{\sin r} = \mu, \quad \text{and} \quad \frac{\sin i'}{\sin r'} = \frac{1}{\mu}, \quad \text{or} \quad \frac{\sin r'}{\sin i'} = \mu;$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{\sin r'}{\sin i'}.$$

But since the normals $BP$ and $CQ$ are parallel, the alternate angles $r$ and $i'$ are equal; and, as the denominators in the above equation are equal, the numerators must also be equal, and therefore the angles $r'$ and $i$ are equal to one another, and hence $CD$ is parallel to $AB$. Hence, when a ray of light passes through a transparent plate with parallel faces, *the emergent ray is parallel to the original direction of the ray before entering.*

**Cor.**—It can be shown in the same way that if a ray passes in succession through a number of plates with parallel faces, some of which are repeated, its directions in the same medium are always parallel. For example, in fig. 26 is represented a number of plates; then the paths of the rays in 1 and 6 will be parallel to one another, as also those in 2 and 4, and in 3 and 5.

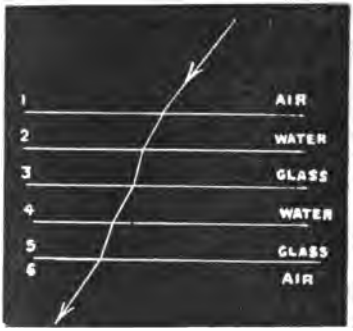

FIG. 26.

### *To find the relation between the true and apparent depths of an object under water or other liquid.*

35. Let $A$ be a luminous point at a depth $D$ below the surface of the water or other liquid, and let a pencil of rays

proceed from $A$ of which $AOB$ is the path of any one ray, being bent from the normal $OP$ on passing out into the air.

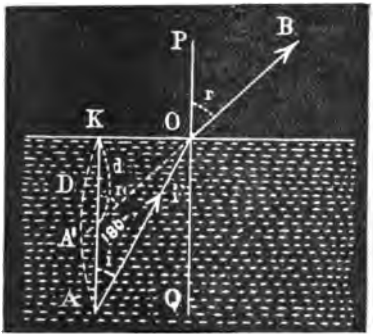

FIG. 27.

The angle $AOQ$ is now the angle of incidence $i$, and $BOP$ the angle of refraction $r$. Produce $OB$ back to meet the perpendicular $OK$ in $A'$, and let $A'K = d$.

Since the normal $PQ$ is parallel to $AK$, the angle $OAK = i$, and the angle $OA'K = r$.

Hence $\sin i = \dfrac{KO}{AO}$, and $\sin r = \dfrac{KO}{A'O}$;

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{KO}{AO} \times \frac{A'O}{KO} = \frac{A'O}{AO};$$

but, since the ray now passes from the water to the air,

$$\frac{\sin i}{\sin r} = \frac{1}{\mu}; \quad \therefore \quad \frac{A'O}{AO} = \frac{1}{\mu}, \quad \text{or} \quad AO = \mu A'O.$$

If now we imagine the rays of the pencil to fall almost perpendicularly on the surface so that $O$ and $K$ are very close together, then

$AO$ becomes $= AK = D$, and $A'O = A'K = d$.

Hence $D = \mu d$,

or, real depth = $\mu$ × apparent depth.

36. Since $\mu$ for any liquid is greater than unity, an object in a liquid will not appear to be as deep as it really is. For example, since $\mu = \frac{4}{3}$ for water, it follows that if water appears to be 3 feet deep, its real depth is 4 feet; and when a person steps into a pool of water, it is always found to be deeper than it appears to be. This difference is more pronounced in the case of sea-water on account of its greater density. The fact that water appears less deep than is really the case adds somewhat to the dangers of bathing to those unable to swim.

The same law holds for transparent solids: a sheet of glass does not appear, on looking through it, to be as thick as it really is.

37. The following experiment will illustrate how objects immersed in water appear to be at a less depth than is really the case:—

Let *A* be the position of a small coin, such as a sixpence, placed in the bottom of an empty vessel with non-transparent sides, and let *E* be the eye of an observer in such

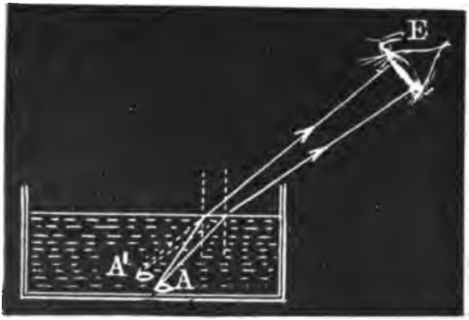

FIG. 28.

a position that the opaque side of the vessel just hides the coin from his view. Let water be now carefully poured into the vessel so as not to displace the coin. The bottom of the vessel will now appear to rise, and the coin will be seen at $A'$. For, a pencil of light proceeding from $A$ is, on emerging from the water, bent from the normal, and enters the observer's eye as if it came from $A'$.

In the same way an oar placed obliquely and partly immersed in water will appear broken, the portion below the water appearing raised by refraction.

Ex. 1. A river appears to be 5 feet deep: find its real depth, the refractive index of water being $\frac{4}{3}$.

Here, $D = \mu d$, or $D = \frac{4}{3} \times 5 = 6' \, 8''$.

Ex. 2. A sheet of glass, on looking through it, appears to be $\frac{1}{2}$ an inch thick: find its real thickness, the refractive index of glass being $\frac{3}{2}$.

Here, $D = \mu d$, or $D = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$ inch.

### EXERCISES IV.

1. If a ray of light pass from air into a substance making an angle of 45° with the normal, find the refractive index of the substance if the ray after refraction makes an angle of 30° with the normal.

2. Find the sine of the angle of refraction of a ray of light incident on the surface of glass at an angle of 60° with the normal, the refractive index of glass being $\frac{3}{2}$.

3. The refractive index of a substance is 2, and if, as a ray of light passes from the substance into air, the angle of incidence be slightly less than 30°, find the angle of refraction, i.e. the angle the ray, as it emerges into the air, makes with the normal. (N.B.—In this case, $\frac{\sin i}{\sin r} = \frac{1}{\mu}$, since the ray passes from the substance into air.)

4. On consulting a table of refractive indices, it is found that $\mu_1$ is the refractive index for a ray passing from air into a substance $A$, and $\mu_2$ the index for a substance $B$. Hence calculate the refractive index for a ray passing from $A$ into $B$.

5. Find the refractive index for a ray passing from water into glass, the refractive indices of water and glass being $\frac{4}{3}$ and $\frac{3}{2}$ respectively.

6. The bottom of a vessel filled with water appears to be 7 feet below the surface: what is the real depth of the water?

7. On looking down into a beaker of sulphuric acid the liquid appears to be 10″ in depth: find the real depth, the refractive index of the acid being 1·434.

8. Prove that, when a ray of light passes through a transparent plate with parallel faces, it emerges parallel to its original direction.

9. Hence prove that, when a ray passes in succession through a number of plates with parallel faces—say $M_1$, $M_2$, and $M_3$ of different degrees of refrangibility—the total deviation of the ray is the same as if it passed through the last plate $M_3$ alone without the intervention of the others.

10. Hence show that, assuming the Earth to be a plane, and the different superincumbent layers of atmosphere of varying density to be horizontal planes with parallel faces, the total deviation of a ray of light from a star passing through these layers is the same as if it passed through a homogeneous atmosphere of the same density and refrangibility as the lowest layer.*

### *The Total Reflection of Light.*

38. We have seen that when a ray of light passes from a rarer into a denser medium, as from air into water, the

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* This is a very important principle in the theory of Astronomical Refraction. See "Elements of Astronomy," chap. iv.

angle of incidence is greater than the angle of refraction, the relation between them being given by the equation

$$\frac{\sin i}{\sin r} = \mu.$$

Since $\mu$ is a constant, it follows that as the value of $i$ increases, that of $r$ also increases; so that when the angle of incidence $i$ is increased to nearly 90°, the angle of refraction $r$ reaches its maximum value. Let $x$ be this maximum value of $r$, the value of $x$ being thus found:—

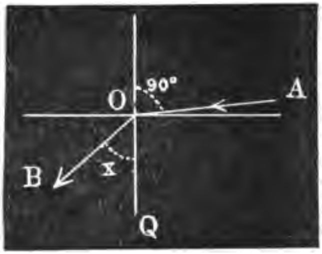

FIG. 29.

$$\frac{\sin 90}{\sin x} = \mu, \quad \text{or} \quad \frac{1}{\sin x} = \mu, \quad \text{or} \quad \sin x = \frac{1}{\mu}.$$

We thus see that a ray of light can pass from a rarer to a denser medium for all angles of incidence from 0° to 90°; but in no case can the refracted ray lie outside the angle $x$ determined by the above equation.

39. **Total Reflection. Critical Angle.**—This is not the case, however, when the ray is reversed and passes from the more refractive to the less refractive medium, as from water into air. For, let *ABQ*, *CDQ*, and *FOQ* (fig. 30) be the paths of rays to the point *Q* from the rarer into the denser medium, *FOQ* being the particular ray where the angle of incidence is almost 90°, and therefore where the angle of refraction reaches its maximum value $x$. Conversely, *QBA*, *QDC*, and *QOF* represent the

paths of rays from *Q* passing from the denser into the rarer medium, and *QO* will be the particular ray from *Q* where the emergent ray *OF* is almost coincident with the bounding surface. The angle *x* between the incident ray *QO* and the normal, when the ray emerges almost coincident with the bounding surface, is called the *critical angle*;

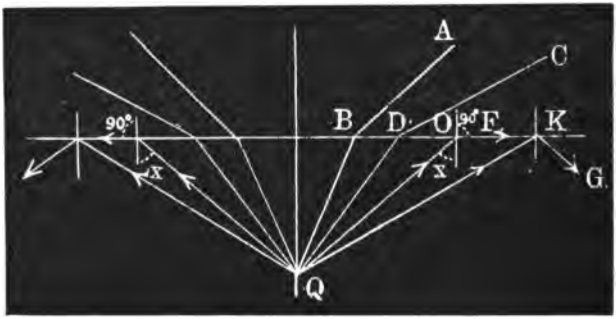

FIG. 30.

for, as we have seen, all incident rays such as *QB* and *QD*, where the angle made with the normal is less than the critical angle, are able to emerge into the rarer medium; but when a ray *QK* is incident in the more refractive medium at an angle greater than the critical angle, it can be shown by experiment that this ray is reflected along *KG*, the angle of reflection being equal to the angle of incidence, as if the bounding surface were a highly polished mirror. This reflection is nearly perfect, and is called *total reflection*, and takes place *when the rays are incident in the more refractive medium at an angle greater than the critical angle.*

The critical angle is sometimes called *the angle of total reflection,* and its value is given, as we have seen above, by the equation

$$\sin x = \frac{1}{\mu}.$$

That is, *the sine of the critical angle is equal to the reciprocal of the index of refraction of the two media* when the light is supposed to pass from the rarer into the denser medium.

For water, $\sin x = \frac{3}{4}$, and $x = 48^\circ\ 35'$.
For glass, $\sin x = \frac{2}{3}$, and $x = 41^\circ\ 48'$.

40. The phenomenon of the total reflection of light may be observed in many instances which are matters of

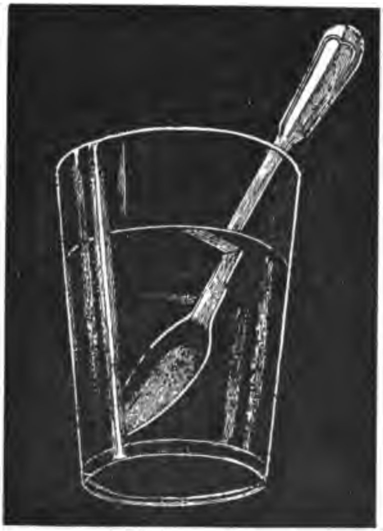

FIG. 31.

daily experience. For example, if a tumbler of water with a spoon in it (fig. 31) be taken and held above the level of the eye, it will be seen, on looking up, that the under surface of the water shines like a brilliant mirror, and the reflection of the lower part of the spoon is seen just as in a plane mirror.

Ex. 1. If the refractive index of a substance be 2, show that when a ray from a point in the substance is incident on an external plane surface of the substance, the ray will emerge into the air, just graze along the plane surface, or be reflected back again into the body, according as the angle of incidence is less than, equal to, or greater than 30°.

Here, $\sin x = \frac{1}{\mu} = \frac{1}{2}$; $\therefore$ the critical angle $x = 30°$, from which above results follow by Art. 38.

Ex. 2. The index of refraction of light passing from air into glass being $\frac{3}{2}$, and from air into water $\frac{4}{3}$, find the index of refraction from water to glass. At a surface separating glass from water, in which medium can total reflection take place, and what will be the sine of the critical angle?

Here the index from water to glass $= \frac{3}{2} \div \frac{4}{3} = \frac{9}{8}$. (See Ex. 4, page 46.)

Total reflection can only take place in the medium with the greater index of refraction, viz. the glass (Art. 38).

Also, if $x$ be the critical angle, and $\mu$ the index of refraction from water to glass,

$$\sin x = \frac{1}{\mu} = \frac{1}{\frac{9}{8}} = \frac{8}{9}.$$

41. **The Prism.**—In Optics, a *prism* is a transparent

body bounded by two plane faces inclined to one another. The line in which these two plane faces meet is called the *edge* of the prism, the angle between the two planes the *refracting angle*, and a plane section of the prism perpendicular to its edge is called a *principal section*. However, the prisms usually used in optical experiments are also prisms in the geometrical sense of the word; their principal sections are triangles, as seen in fig. 32, in which a prism and a principal section are shown.

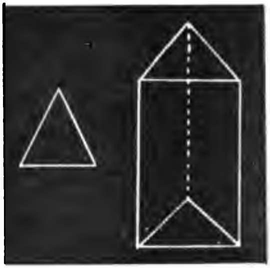

FIG. 32.

### *To find the deviation of a Ray of Light incident nearly perpendicularly upon a Prism of small angle.*

42. Let $PQR$ be a principal section of a thin prism

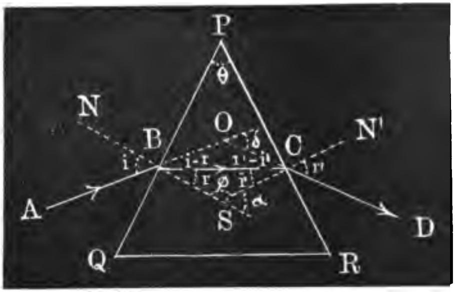

FIG. 33.

where the refracting angle $\theta$ is supposed very small (though drawn large in diagram for greater clearness).

Let $ABCD$ be the path of a ray falling almost perpendicularly on the prism, being bent along $BC$, towards the normal $NS$ on entering, and along $CD$ from the normal $N'S'$ on leaving, the prism. Produce $AB$ and $CD$ to meet at $O$; then the external angle $\delta$ at $O$, being the angle between the original direction $AB$ and its final direction $CD$, will be the *deviation.*

Let $i$ and $r$ be the angles of incidence and refraction on entering the prism, and $i'$ and $r'$ the angles of incidence and refraction on leaving.

Hence the angle $OBC = i - r$,

and the angle $OCB = r' - i'$.

Also, since the angles $PBS$ and $PCS$ are right angles, the quadrilateral $PBSC$ is inscribable in a circle, and therefore

$$\angle\,\theta + \angle\,\phi = 180^\circ;$$

but

$$\angle\,a + \angle\,\phi = 180^\circ,$$

therefore the angles $a$ and $\theta$ are equal to one another.

Again (*Euclid* I, 32),

$$\delta = OBC + OCB,$$

or

$$\delta = i - r + r' - i'.$$

But

$$\frac{\sin i}{\sin r} = \mu, \quad \text{and} \quad \frac{\sin i'}{\sin r'} = \frac{1}{\mu};$$

and, since the incident ray $AB$ strikes the prism almost perpendicularly, the angles $i$, $r$, $i'$, and $r'$ are very small, and their sines are almost equal to the angles themselves expressed in circular measure.

Hence $\frac{i}{r} = \mu, \quad \text{and} \quad \frac{i'}{r'} = \frac{1}{\mu};$

therefore $i = \mu r, \quad \text{and} \quad r' = \mu i'.$

Hence
$$\begin{aligned}\delta &= \mu r - r + \mu i' - i' \\ &= r(\mu - 1) + i'(\mu - 1) \\ &= (\mu - 1)(r + i').\end{aligned}$$

But $r + i' = a = \theta$ (*Euclid* I, 32).

Hence $\delta = (\mu - 1)\theta.$

From which we conclude that *when the refracting angle of a prism is small, and the incident ray is nearly perpendicular to the prism, the deviation of the ray is constant, as it depends only on $\mu$, the index of refraction of the substance of which the prism is composed, and $\theta$, the refracting angle of the prism.*

From the above figure it is seen that the ray $AB$ is, in its passage through the prism, *bent away from the refracting angle at $P$*; and hence, if an object at $A$ be viewed from $D$, the rays enter the eye in direction $OCD$, *the image of the object being moved towards the refracting angle.* The student can easily verify this result by looking at any object through an angular piece of glass, when it is seen that the image of the object appears displaced towards the apex of the angle.

Ex. 1. If the angle of a glass prism be 48′, find the deviation of a ray of light passing almost perpendicularly through it.

Here,
$$\delta = (\mu - 1)\theta = (\tfrac{3}{2} - 1).48' = 24'.$$

**Ex. 2.** If a ray, incident almost perpendicularly on the surface of a prism of fluor-spar, be deviated through an angle of 14′, find the refractive angle of the prism, the refractive index of fluor-spar being 1·434.

Here, $\delta = (\mu - 1)\,\theta$, or $14' = (1{\cdot}434 - 1)\,\theta$.

Hence
$$\theta = \frac{14'}{{\cdot}434} = 32'\,15''.$$

**Ex. 3.** Find the angle of a water prism which will produce a deviation equal to that produced in a glass prism whose angle is 1° 20′.

For the glass prism,
$$\delta = (\mu - 1)\,\theta, \quad \text{or} \quad \delta = (\tfrac{3}{2} - 1)\,80' = 40'.$$

For the water prism,
$$\delta = (\mu - 1)\,\theta, \quad \text{or} \quad 40' = (\tfrac{4}{3} - 1)\,\theta; \quad \therefore\ \theta = 120' = 2^\circ.$$

**Note.**—In the following Exercises, except in the case of example 12, when the light passes through a prism, the incident rays are supposed to fall almost perpendicularly on its surface.

## EXERCISES V.

1. What is meant by the total reflection of light, and critical angle? and show how the value of the latter can be calculated for any transparent substance.

2. If the refractive index of a substance be $\sqrt{2}$, show that an incident ray in the substance will emerge into the air, just graze along the surface, or be reflected back again into the substance, according as the angle of incidence is less than, equal to, or greater than 45°.

3. The index of refraction from air to water is $\frac{4}{3}$, and from air to diamond, 2·44: find the index of refraction from water to diamond. At a surface separating water from diamond, in which medium can total reflection take place, and what will be the sine of the critical angle?

4. A stone is placed at the bottom of a large vessel filled with water. The bottom of the vessel is transparent. On looking through the bottom, the stone can be seen by total internal reflection at the upper surface of the water, if the eye is some distance from the stone, but not if it is close to it. Explain.

5. Find the sine of the angle of total reflection of ruby whose refractive index is 1·78.

6. Find the sine of the critical angle for alcohol whose refractive index is 1·37.

7. If the angle of a water prism be 1° 30′, find the deviation of a ray of light incident on its surface at a small angle.

8. Find the angle of a glass prism which will produce a deviation equal to that produced in a water prism whose angle is 54′.

9. If the deviation produced by a prism of fluor-spar be 34′ 43″, and the refracting angle of the prism 1° 20′, find the refractive index of fluor-spar.

10. The deviation produced by a thin prism is one-third of the angle of the prism: find the index of refraction.

11. A prism of glass whose angle is 1° 40′ is surrounded with water: find the deviation of a ray of light passing almost perpendicularly through the prism.

12. Prove that for any prism, when a ray of light passes through it so as to make equal angles with the two faces,

$$\mu = \frac{\sin\frac{\delta+\theta}{2}}{\sin\frac{\theta}{2}};$$

and hence that the index of refraction of the material of which any prism is composed can be found, if the angle $\theta$ of the prism and the deviation $\delta$ of a ray making equal angles with its two faces be known.

## *Lenses.*

43. **Def.**—A *lens* is a transparent body bounded by two spherical surfaces.

These surfaces may either be both portions of spheres of finite radii, or one of these surfaces may be a plane—a plane being regarded as a sphere of infinite radius.

**Convex or Converging Lenses.**—A convex lens may have one or other of the three forms *A*, *B*, or *C* in fig. 34. *A* is a double convex, *B* a plano-convex, and *C* a concavo-convex lens, or a converging meniscus. All these convex lenses are thicker in the middle than at the edges, and they are all converging lenses, as will be shown in Art. 44, because rays passing through them are more convergent or less divergent than before.

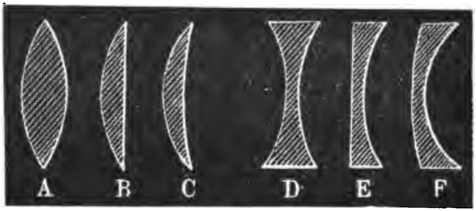

FIG. 34.

**Concave or Diverging Lenses.**—A concave lens may have one or other of the three forms *D*, *E*, or *F* in fig. 34. *D* is a double concave, *E* a plano-concave, and *F* a convexo-concave lens, or a diverging meniscus. All these concave lenses are thinner in the middle than at the edges, and they are all diverging lenses, as will be shown in Art. 44, because rays passing through them emerge more divergent or less convergent than before.

44. **Def.**—The line joining the centres of the two spherical surfaces is called the *axis* of the lens.

That convex lenses are converging, bending a ray of light passing through them towards the axis, is seen from fig. 35 (*a*), which represents the path *ABCD* of a ray passing through a convex lens. For, at the points *B* and *C* draw tangent planes meeting in *P* and forming a prism.

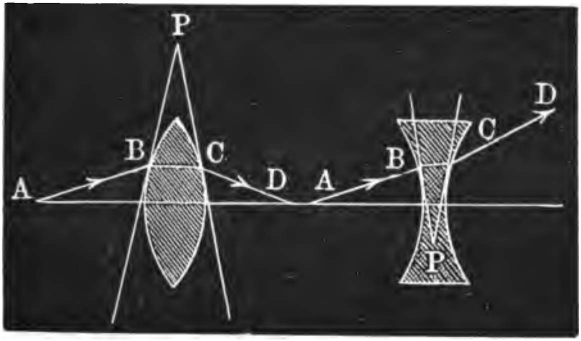

Fig. 35 (*a*). Fig. 35 (*b*).

The path of the ray through the lens will be the same as through this prism, which, as its vertex *P* points upwards, bends the ray away from *P*, and therefore towards the axis (Art. 42).

Again, in the case of concave lenses (fig. 35 (*b*)), the tangent planes at *B* and *C* form a prism whose vertex *P* is downwards; and hence the ray *BCD* will be bent upwards (Art. 42), and therefore away from the axis; and hence these lenses are diverging.

**Note.**—A lens is said to be *thin* when its thickness is small compared with each of the radii of curvature of its bounding surfaces.

### *To find the deviation of a Ray of Light in passing nearly perpendicularly through a thin lens.*

45. **Case I. Concave Lenses.**—Let *ABKE* be the path of a ray of light passing through the convexo-concave lens in fig. 36, as if it passed through a prism

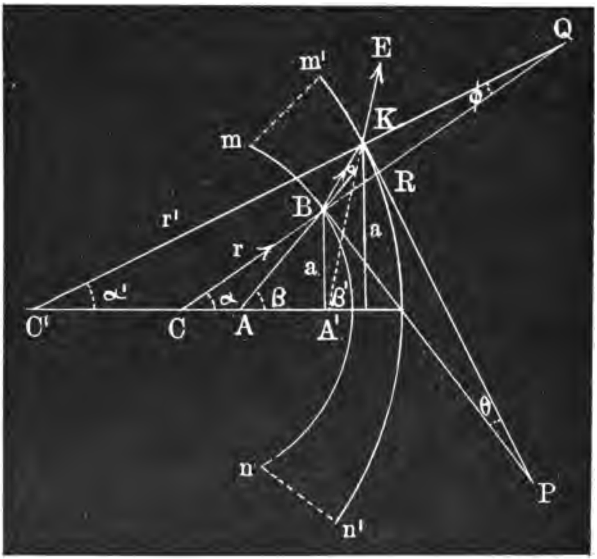

FIG. 36.

whose vertex is at *P*, formed by drawing tangent planes at *B* and *K*. Let *C* be the centre of curvature of that surface *mn* which is nearer the incident light, and *C′* the centre of curvature of the surface $m'n'$ which is more remote.

$r$ = radius of curvature of the surface $mn$ next the incident light;

$r'$ = radius of curvature of the surface $m'n'$ remote from the incident light;

$a$ = distance of $B$ or $K$ from the axis (the lens being thin, these two distances are approximately equal).

$\delta$ = deviation.

Now $\delta = (\mu - 1)\,\theta$. (Art. 42.)

But since, in the two triangles $PBR$ and $QKR$ the opposite angles at $R$ are equal, and the angles at $B$ and $K$ are both right, therefore the remaining angles $\theta$ and $\phi$ are equal to one another.

$$\therefore \quad \delta = (\mu - 1)\phi = (\mu - 1)(\alpha - \alpha').$$

But $\alpha$ and $\alpha'$ are small angles; expressing them in circular measure, we may in each case regard $a$ as an arc.

$$\therefore \quad \alpha = \frac{a}{r}, \quad \text{and} \quad \alpha = \frac{a}{r'};$$

$$\therefore \quad \delta = (\mu - 1)\left(\frac{a}{r} - \frac{a}{r'}\right),$$

or

$$\delta = a\,(\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

46. **Case II. Convex Lenses.**—Here the ray passes through the lens (fig. 37) along the path $ABKA'$, as if through the prism whose angle is $\theta$. The lettering is the same as before. Also the angles $\theta$ and $\phi$ are equal

(Art. 41), and $r$, being measured away from the incident light, is negative.

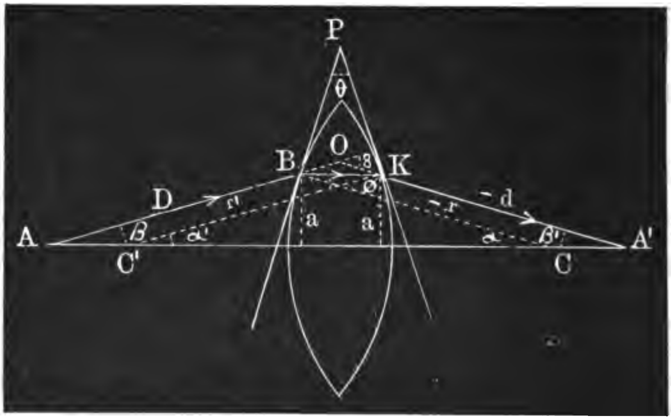

FIG. 37.

As before, we have $\delta = (\mu - 1)\theta$. (Art. 42.)
But $\theta = \phi = \alpha + \alpha'$;
$\therefore \delta = (\mu - 1)(\alpha + \alpha')$.

But $\alpha$ and $\alpha'$ being small angles,

$$\alpha = \frac{a}{-r}, \quad \text{and} \quad \alpha' = \frac{a}{r'}.$$

Also $\delta$ in convex lenses is negative; for the ray tends, after refraction, to converge towards the axis, and $\delta$ is therefore downwards, whereas it would be measured upwards, and therefore positive, for a concave lens.

$$\therefore \quad -\delta = (\mu - 1)\left(\frac{a}{-r} + \frac{a}{r'}\right),$$

or

$$-\delta = a(\mu - 1)\left(-\frac{1}{r} + \frac{1}{r'}\right),$$

or

$$\delta = a(\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right),$$

the same formula as before, due regard being given to the signs of the lines, any distance being considered + or −, according as it is measured *towards* or *from* the incident light.

If we assume the light to be moving in the opposite direction, the path of the ray being *A'KBA*, then we must change $r$ to $-r'$ and $r'$ to $-r$, and we get the same result as before; this is also true of the concave lens in Art. 45, and hence the above formula is true in all cases.

47. From the above formula it follows that the deviation of a ray of light passing almost perpendicularly through a thin lens is directly proportional to the distance $a$ of the point at which it meets the lens from the axis, and is therefore constant when the distance of this point from the axis is constant.

## *Conjugate Foci of Thin Lenses.*

48. **Case I. Concave Lenses.**—Let *ABKE* be the path of a ray proceeding from a radiant point *A* (fig. 36). Produce *KE* backwards to meet the axis in *A'*. Then if *AB* and *KE* meet at *O*, the acute angle at *O* is $\delta$, the deviation (Art. 45).

Let $AB = D$, and $A'K = d$, both being positive.

Now, as in Art. 45, we have

$$\delta = (\mu - 1)\theta = (\mu - 1)\phi, \quad \text{for} \quad \theta = \phi;$$

but

$$\delta = \beta' - \beta, \quad \text{and} \quad \phi = \alpha - \alpha';$$

$$\therefore \quad \beta' - \beta = (\mu - 1)(\alpha - \alpha').$$

All the angles being small when expressed in circular measure, we have

$$\alpha = \frac{a}{r}, \quad \alpha' = \frac{a}{r'}, \quad \beta = \frac{a}{D}, \quad \text{and} \quad \beta' = \frac{a}{d};$$

$$\therefore \quad \frac{a}{d} - \frac{a}{D} = \left(\mu - 1\right)\left(\frac{a}{r} - \frac{a}{r'}\right),$$

or

$$\frac{1}{d} - \frac{1}{D} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

As this equation is independent of $a$, the distance of the point at which the ray meets the lens from the axis, it follows that all rays proceeding from $A$, and falling almost perpendicularly on the lens, will, after refraction, when produced, meet in a conjugate focus $A'$; or, in other words, to every point $A$ on the axis of a thin lens there is a corresponding conjugate focus.

49. **Case II. Convex Lenses.**—Similarly, in the case of a convex lens (fig. 37), the path of the ray proceeding from a radiant point $A$ is along $ABKA'$. Let $AB = D$, and $A'K = -d$ (being negative as it is measured from the incident light).

As before, $\delta = (\mu - 1)\,\theta = (\mu - 1)\,\phi$;

but $\delta = \beta + \beta'$, and $\phi = \alpha + \alpha'$;

therefore $\beta + \beta' = (\mu - 1)\,(\alpha + \alpha')$.

And, as before,

$$\frac{a}{D} + \frac{a}{-d} = \left(\mu - 1\right)\left(\frac{a}{-r} + \frac{a}{r'}\right),$$

or

$$\frac{1}{D} - \frac{1}{d} = \left(\mu - 1\right)\left(-\frac{1}{r} + \frac{1}{r'}\right),$$

or

$$\frac{1}{d} - \frac{1}{D} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right);$$

the same result as before.

As before, this result is independent of $a$; and hence all rays proceeding from $A$, and falling almost perpendicularly on the lens, will be refracted to a conjugate focus $A'$.

If, in both above cases, we imagine the path of the ray to be reversed, then change $r'$ to $-r$ and $r$ to $-r'$, and we see that the formula remains unchanged, and therefore is universally true whatever be the direction of the ray.

Ex. 1. The radii of a convexo-concave lens of glass are 6″ and 8″ respectively. A pencil of light radiates from a point situated 1 foot from the lens on the same side of it as the centres of curvature lie: find the conjugate focus.

Here (fig. 36), $\frac{1}{d} - \frac{1}{D} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right)$,

$D = AB = 12''$, $r =$ radius of $mn = 6''$, $r' =$ radius of $m'n' = 8''$; both radii being positive, being measured towards the incident light.

$$\therefore \quad \frac{1}{d} - \frac{1}{12} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{6} - \frac{1}{8}\right) = \frac{1}{2} \times \frac{1}{24} = \frac{1}{48};$$

$$\therefore \quad \frac{1}{d} = \frac{1}{12} + \frac{1}{48} = \frac{5}{48}; \quad \therefore \; d = 9{\cdot}6'' \text{ in front of lens.}$$

Ex. 2. A double convex lens of glass has radii 6″ and 10″ respectively. If the radiant point be situated 16″ from the lens, find the position of the conjugate focus.

Here (fig. 37), $\frac{1}{d} - \frac{1}{D} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right)$,

$D = AB = 16''$, $r =$ radius of curve at $B = -6''$,

$r' =$ radius of curve at $K = 10''$;

$$\therefore \quad \frac{1}{d} - \frac{1}{16} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-6} - \frac{1}{10}\right) = -\frac{2}{15};$$

$$\therefore \quad d = -14{\cdot}12''.$$

Hence the refracted pencil *converges* to a point 14·12″ beyond the lens.

**Ex. 3.** A double concave lens of glass has radii 10″ and 12″ respectively, and the distance of the incident focus is 30″: find the conjugate focus.

Here $r = 10$ and $r' = -12$, and

$$\frac{1}{d} - \frac{1}{30} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{-12}\right) = \frac{11}{120};$$

$$\therefore \quad d = 8''.$$

Hence the pencil *diverges* as if coming from a point 8″ in front of the lens.

**Ex. 4.** A radiant point is situated at a distance of 24″ from a plano-convex lens of fluor-spar (refractive index = 1·434), the radius of whose spherical surface is 12″: find the position of the conjugate focus.

Here $r = -12''$, and $r' = \infty$, and $\frac{1}{\infty} = 0$, and

$$\frac{1}{d} - \frac{1}{24} = \left(1{\cdot}434 - 1\right)\left(-\frac{1}{12}\right);$$

$$\therefore \quad d = 181{\cdot}8''.$$

Hence the pencil *diverges*, after refraction, as if coming from a point 181·8″ situated on the same side of the lens as the given radiant point.

**Ex. 5.** In last question, the curved surface whose radius was 12″ was taken closest to the incident light: show that the result would have been the same if the lens were reversed.

Here $r = \infty$, and $r' = +12$, and

$$\frac{1}{d} - \frac{1}{24} = \left(1{\cdot}434 - 1\right)\left(\frac{1}{\infty} - \frac{1}{12}\right) = \left(1{\cdot}434 - 1\right)\left(-\frac{1}{12}\right),$$

and $d = 181{\cdot}8''$, as before.

## EXERCISES VI.

1. A radiant point is situated at a distance of 20 inches from a plano-convex lens of glass ($\mu = \frac{3}{2}$), the radius of whose spherical surface is 10″: find the position of the conjugate focus.

2. In last example, if the lens be plano-concave, find the conjugate focus.

3. A double convex lens of glass has radii of 8″ and 12″ respectively: find the conjugate focus of a radiant point situated from it at a distance of 16″.

4. The radius of the curved surface of a plano-concave lens of glass is 12″, and a radiant point is situated at a distance of 18″ from it: find the position of the conjugate focus.

5. A radiant point is situated at a distance of 8″ from a double concave lens of glass whose radii are 12″ and 16″ respectively: find the conjugate focus.

6. If, in Example 5, the light *converges* to a point 8″ *behind* the lens, find the conjugate focus. (Here, $D = -8''$.)

7. Let the radii of a converging meniscus be 10 and 12 inches respectively, and the radiant point be 60″ from the lens: find the conjugate focus.

8. If, in Example 7, the light converged to a point 20″ behind the meniscus, find the conjugate focus.

50. **Principal Focus. Focal Length.**—If, in Arts. 48, 49, we suppose the incident rays to be parallel to the axis of the lens, then the radiant point $A$ will be at an infinite distance from the lens, and those parallel rays which are close to the axis of the lens will be brought to a focus on the axis called the *principal focus*, and the distance of this principal focus from the lens is called the *focal length* of the lens.

In this case, $D = \infty$, and therefore $\frac{1}{D} = 0.$

Also, $d$ becomes the focal length $f$ of the lens, and the formula

$$\frac{1}{d} - \frac{1}{D} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

becomes

$$\frac{1}{f} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

51. **Real and Virtual Principal Foci.**—A convex lens will converge parallel rays towards a point $F$ (fig. 38) on the axis on the other side of the lens. As the rays, after refraction, actually pass through $F$, it follows that the principal focus of a convex lens is *real*, and its focal length is negative, being on the opposite side of lens. In the case of a concave lens, parallel rays are diverged from the axis, and emerge as if they had come from the principal focus $F$ (fig. 39). In this case, the focus $F$ is *virtual*, and the focal length is positive, as the principal focus is on the same side of the lens as the incident light.

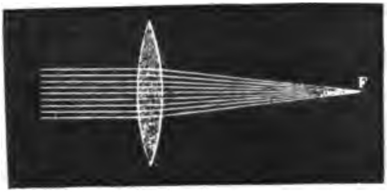

**Fig. 38.**

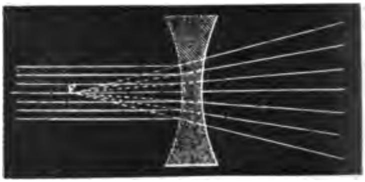

**Fig. 39.**

If the parallel rays came from the opposite direction, then it would be found that each lens had another principal focus the same distance on the other side.

Ex. 1. Find the principal focus of a double convex lens of ice of radii 8″ and 10″ respectively, the index of refraction of ice being 1·31.

Here, $$r = -8'', \quad r' = 10'',$$

and $$\frac{1}{f} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right),$$

or $$\frac{1}{f} = \left(1{\cdot}31 - 1\right)\left(-\frac{1}{8} - \frac{1}{10}\right) = {\cdot}31\left(-\frac{9}{40}\right);$$

$$\therefore \quad f = -14{\cdot}34''.$$

Hence the principal focus is 14·34″ behind the lens.

Ex. 2. Find the position of the principal focus of a double convex lens of glass of equal curvature.

Here, $$\frac{1}{f} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

But $r$ is negative, and $r' = r$, the curvatures being equal;

$$\therefore \quad \frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(-\frac{1}{r} - \frac{1}{r}\right) = \frac{1}{2} \times -\frac{2}{r} = -\frac{1}{r};$$

$$\therefore \quad f = -r.$$

Hence the principal focus is at the centre of the first curve which the incident light meets.

## EXERCISES VII.

1. Find the principal focus of a double concave lens of rock-salt, the radii of curvature of whose surfaces are 6″ and 8″ respectively, the refractive index of rock-salt being 1·557.

2. Find the focal length of a double concave lens (*a*) of glass, (*b*) of water, of equal curvature.

3. Find the principal focus of a double convex lens of water of equal curvature.

4. Find the principal focus of a plano-convex lens (*a*) of glass, (*b*) of water.

5. Find the principal focus of a plano-concave lens (*a*) of glass, (*b*) of water.

6. Find the focal length of a double convex lens of fluor spar, each radius of curvature being 10″, and the index of refraction 1·434.

7. Light moving parallel to the axis of a concavo-convex lens, the radii of curvature of whose faces are 12″ and 6″, and for which the index of refraction is $\frac{3}{2}$, passes through the lens: what will happen to the light?

### *To find the Focal Length of a Lens in terms of the distances of any two conjugate foci from the lens.*

52. On comparing the two formulæ in Arts. 49, 50, it follows that $\frac{1}{d} - \frac{1}{D}$ and $\frac{1}{f}$, being each equal to

$$\left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right),$$

are therefore equal to one another. Hence we have

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f},$$

where $D$ and $d$ are the distances of any two conjugate foci from a thin lens of focal length $f$.*

From this equation the relative positions of two conjugate foci can, in all cases, be deduced thus:—

53. **Concave Lenses.**—Here we have

$$\frac{1}{d} = \frac{1}{D} + \frac{1}{f},$$

and, since $f$ is positive (Art. 51), it follows that $d$ is positive, and therefore the two conjugate foci are situated on the same side of the lens.

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* A direct geometrical proof of this formula will be found in Art. 61.

54. **Convex Lenses. Case I.**—Let the radiant point be situated at a greater distance from the lens than its focal length. Then, since $f$ is negative (Art. 51), we have

$$\frac{1}{d} = \frac{1}{D} - \frac{1}{f};$$

but $$D \text{ is } > f, \quad \therefore \frac{1}{D} \text{ is } < \frac{1}{f};$$

therefore $\frac{1}{d}$ is negative, and hence $d$ is negative, and the conjugate focus is therefore behind the lens, the two conjugate foci lying on opposite sides of the lens.

**Case II.**—Where the radiant point is between the principal focus and the lens.

As before, we have

$$\frac{1}{d} = \frac{1}{D} - \frac{1}{f};$$

but $$D \text{ is } < f, \quad \therefore \frac{1}{D} \text{ is } > \frac{1}{f};$$

therefore $\frac{1}{d}$ is positive, and < than $\frac{1}{D}$.

Hence $d$ is positive and $> D$, and hence the conjugate focus is on the same side of the lens as the radiant point, and at a greater distance from the lens.

**Note.**—In applying the above equation to numerical examples, the same rule as regards the signs of the quantities must be followed as in the other formulæ—*viz.*, that any distance is to be considered plus or minus, according as it is on the same or on the opposite side of the lens respectively as the incident light.

Ex. 1. If a radiant point is situated 12″ from a concave lens whose focal length is 10″, find the conjugate focus.

Here, $D = +12$, and $f = +10$;

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f},$$

or $$\frac{1}{d} - \frac{1}{12} = \frac{1}{10}; \quad \therefore \quad \frac{1}{d} = \frac{1}{10} + \frac{1}{12} = \frac{11}{60};$$

$$\therefore \quad d = \frac{60}{11} = 5\tfrac{5}{11}'' \text{ in front of lens.}$$

Ex. 2. A pencil of light *converges* to a point 12″ *behind* a concave lens of focal length 10″: find the conjugate focus.

Here, $D = -12$, and $f = +10$;

$$\frac{1}{d} - \frac{1}{-12} = \frac{1}{10}; \quad \therefore \quad \frac{1}{d} = \frac{1}{10} - \frac{1}{12} = \frac{1}{60}; \quad \therefore \quad d = 60'' \text{ in front.}$$

Ex. 3. If a pencil *diverges* from a point 12″ from a convex lens of focal length 10″, find the conjugate focus.

Here, $D = +12''$, and $f = -10''$;

$$\frac{1}{d} - \frac{1}{12} = -\frac{1}{10}; \quad \therefore \quad \frac{1}{d} = \frac{1}{12} - \frac{1}{10}; \quad \therefore \quad d = -60'' \text{ behind the lens.}$$

Ex. 4. Light *converges* to a point 12″ *behind* a convex lens of focal length 10″: find the conjugate focus.

Here, $D = -12''$, and $f = -10''$;

$$\frac{1}{d} + \frac{1}{12} = -\frac{1}{10}; \quad \therefore \quad \frac{1}{d} = -\frac{1}{10} - \frac{1}{12}; \quad \therefore \quad d = -5\tfrac{5}{11}'' \text{ behind the lens.}$$

Ex. 5. The distances of a radiant point and its conjugate focus from a lens are 10″ and 20″ respectively, and both at the same side of the lens: find the focal length, and the nature of the lens.

Here $D = +10$, and $d = +20$;

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f}; \quad \therefore \quad \frac{1}{20} - \frac{1}{10} = \frac{1}{f}; \quad \therefore f = -20'',$$

and the lens is convex, as its focal length is negative.

## EXERCISES VIII.

1. Light diverges from a point 12″ from a concave lens of focal length 18″: find the conjugate focus.

2. Light diverging from a radiant point converges, after refraction, to a point 60″ behind a convex lens of focal length 15″: where must the radiant point have been situated?

3. If in Example 2 the radiant point had been situated 10″ from the lens, would the rays be convergent or divergent after refraction, and where would the conjugate focus be situated?

4. A pencil of light converges to a point situated 6″ behind a convex len of focal length 8″: find the conjugate focus.

5. Two convex lenses *A* and *B*, whose focal lengths are 2″ and 4″ respectively, are 9″ apart; if a radiant point be situated in front of *A* (that is, on the side remote from *B*) and 3″ from it, find the position of the conjugate focus with reference to the combination of lenses.

First, find the conjugate focus of the radiant point with reference to *A*, thus:—

$$\frac{1}{d} - \frac{1}{3} = -\frac{1}{2}; \quad \therefore\ d = -6,$$

*i.e.* 6″ behind *A*, or 3″ in front of *B*. The rays now diverge from a point 3″ in front of *B*.

Again, $$\frac{1}{d} - \frac{1}{3} = -\frac{1}{4}; \quad \therefore\ d = +12,$$

*i.e.* 12″ in front of *B*, or 3″ in front of *A*. Hence the conjugate focus coincides with the original radiant point.

6. A convex and a concave lens, whose focal lengths are 8″ and 10″ respectively, are placed in contact: find the position of the conjugate focus of a radiant point situated 15″ from them.

55. **The Optical Centre of a Lens.**—Let $C$ and $C'$ be the two centres of curvature of a lens. Draw any two parallel radii $CP$ and $C'Q$ to meet the spherical surfaces, and join $PQ$. Let $PQ$ represent the path of a ray entering the lens at $P$, in the direction $AP$, and emerging at $Q$ in the direction $QB$, and cutting the axis in $O$. Then it can be easily shown that the emergent ray $QB$ is parallel to the original direction $AP$.

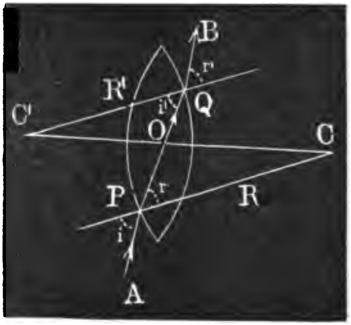

FIG. 40.

For $$\frac{\sin i}{\sin r} = \mu, \quad \text{and} \quad \frac{\sin r'}{\sin i'} = \mu;$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{\sin r'}{\sin i'};$$

but the $\angle r$ = the $\angle i'$ (being alternate angles). Hence the angles $i$ and $r'$ are equal to one another—that is, the ray emerges at the same angle to $C'Q$ as it made at entering with the parallel line $CP$, and therefore $QB$ is parallel to $AP$. The point $O$ found by the above construction is called the centre of the lens. Hence the following definition.

**Def.**—*The centre of a lens is that point through which, when a ray passes, it emerges parallel to its original direction before entering, and therefore suffers no deviation.*

From the similarity of the two triangles $OCP$ and $OC'Q$, we have

$$\frac{OC}{OC'} = \frac{CP}{C'Q} = \frac{R}{R'},$$

where $R$ and $R'$ are the radii of curvature, which shows that the centre of a lens divides the line joining the centres of curvature in the ratio of the radii, and that therefore its position depends only on the ratio of these radii.

It follows from the above that, in the case of a double convex or a double concave lens, the centre is *within* the substance of the lens; in both plano-convex and plano-concave lenses it lies *on* the curved surface; whereas in concavo-convex and convexo-concave lenses it is *outside* the substance of the lens. The student can easily verify these conclusions by drawing a diagram in each case similar to that in fig. 40.

56. **Secondary Axes.**—Any right line passing through the optical centre of a lens other than the principal axis is called a *secondary axis* of the lens. From the above property of the optical centre, it follows that every secondary axis represents the path of a ray passing through this point, for, owing to the lens being very thin, the path of the ray $APQB$ (fig. 40) may be regarded as one continuous straight line.

As long as a secondary axis makes only a small angle with the principal axis, all that has been already stated about the latter is true of the former; that is, rays diverging from any point on such a secondary axis are refracted to a conjugate focus lying on that axis, and determinable by the same formulæ. We shall now make use of this principle in explanation of the formation of images.

### *Geometrical Constructions for determining the position of images in thin lenses.*

57. **Convex Lens. Case I.**—Where the object is placed at a distance from the lens greater than the focal length.

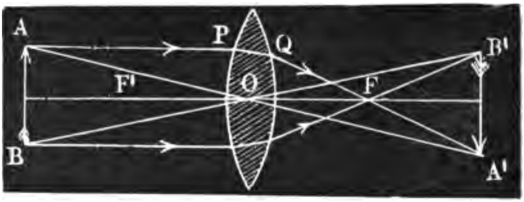

FIG. 41.

Let $AB$ be the object, placed at a distance from the lens greater than the focal length $OF'$. Let $AP$ be the path of a ray parallel to the principal axis. This ray, after refraction, converges along $PQF$ to the principal focus $F$. Hence the conjugate focus of $A$ lies on $QF$. It also lies on the secondary axis $AO$. Hence the conjugate focus of $A$ is at $A'$, where $QF$ and $AO$ intersect. Similarly, the conjugate focus of $B$ is at $B'$, and the same holds for any point intermediate between $A$ and $B$. This image $A'B'$ is *real.* Hence, *when an object is placed outside the principal focus of a convex lens, a real inverted image is formed behind the lens.*

This image, being *real,* can be thrown on a screen placed at $A'B'$, from which it can be viewed by an observer placed in any position from which the front of the screen is visible, or it may be viewed by placing the eye behind $A'B'$ so that the rays emerging from it may fall on the retina.

58. **Size of Image.**—Since the two triangles $OAB$ and $OA'B'$ (fig. 41) are similar to one another, it follows that $AB$ and $A'B'$ are proportional to their distances from the centre of the lens; and if $h$ and $h'$ be the respective heights of $AB$ and of its image $A'B'$, $D$ and $d$ being their respective distances from $O$, we have

$$\frac{h}{h'} = \frac{D}{d}.$$

This formula also applies to the images described in Arts. 59, 60.

59. **Convex Lens. Case II.**—Where the object is placed at a distance from the lens less than its focal length.

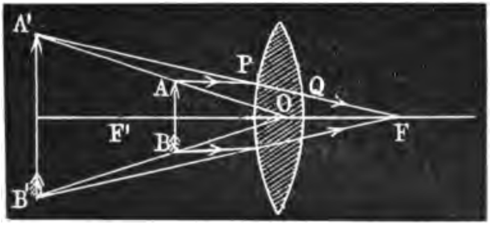

FIG. 42.

Let $AB$ be the object (fig. 42) placed between the principal focus $F'$ and the lens. Then, making the same construction as before, the conjugate focus of $A$ lies on $QF$, and also on $AO$; therefore it must be at their intersection $A'$. Similarly, the focus of $B$ is at $B'$. Hence there is an enlarged image of $AB$ at $A'B'$. This image is *virtual.* Hence, *when an object is placed inside the principal focus of a convex lens, a virtual image is formed, not inverted, and at a greater distance from the lens than the object.*

This image is larger than the object, and the magnified image can be seen when the lens is placed between the object and the eye. Convex lenses are used in this manner as magnifying glasses, and are called *simple microscopes.*

The student will note that in Case I. *AQ* (regarded as a continuous straight line) is greater than *OF*, and hence, as in Art. 26, *AO* and *QF* are convergent, and meet at *A′* on the opposite side of the lens; but in Case II. *AQ* is less than *OF*, and therefore *AO* and *QF*, when produced backwards, meet at *A′* on the same side of the lens as *AB* is situated.

60. **Concave Lens.**—Concave lenses, like convex mirrors, only give virtual images, whatever the distance of the object.

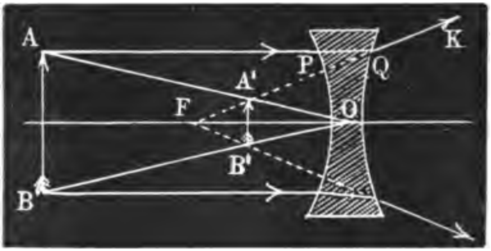

FIG. 43.

In fig. 43 *AB* is the object. A ray *AP* parallel to the axis will be refracted along the path *APQK*, diverging eventually from the axis, as if it came from the principal focus *F*. Hence the conjugate focus of *A* lies on *QF*; and, as it also lies on *AO*, therefore it is at *A′* where *QF* and *AO* intersect. Similarly, the focus of *B* is at *B′*, and hence the image of *AB* is *A′B′*. Therefore, in all cases, *the image of an object formed by a concave lens is on the* *same side of the lens as the object is situated, smaller in size than the object, and erect.*

In all cases the actual position of the image is given by the formulæ already given for finding the position of the conjugate focus of a point, *viz.*:—

$$(1) \qquad \frac{1}{d} - \frac{1}{D} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right),$$

$$(2) \qquad \frac{1}{f} = \left(\mu - 1\right)\left(\frac{1}{r} - \frac{1}{r'}\right),$$

$$(3) \qquad \frac{1}{d} - \frac{1}{D} = \frac{1}{f}.$$

61. We have already proved the formula $\frac{1}{d} - \frac{1}{D} = \frac{1}{f}$ by deduction from the two previous formulæ. It can, however, be proved independently as follows.

*To find the focal length of a lens in terms of the distances of any two conjugate foci from the lens.*

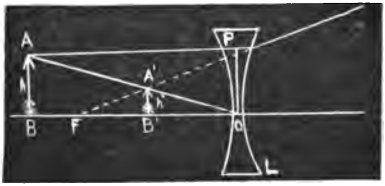

FIG. 44.

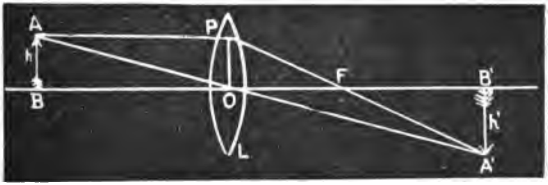

FIG. 45.

Let $AB$ and $A'B'$ (figs. 44, 45) represent an object and its image formed by the lens $L$, the construction for the image being the same as that given in Arts 57–60.

The triangles $ABO$ and $A'B'O$ being similar, we have (*Euclid* vi, 4)

$$\frac{AB}{A'B'} = \frac{BO}{B'O} \quad \text{or} \quad \frac{h}{h'} = \frac{D}{d}.$$

Also the triangles $POF$ and $A'B'F$ are similar;

$$\therefore \quad \frac{PO}{A'B'} = \frac{OF}{B'F}.$$

But $PO = AB = h$, and $OF$ is the focal length; hence, adopting the usual convention as regards signs, we have

$$\frac{h}{h'} = \frac{f}{f-d}; \quad \therefore \quad \frac{D}{d} = \frac{f}{f-d},$$

or $$Df - Dd = df,$$

or $$Df - df = Dd;$$

hence, dividing by $Ddf$, we have

$$\frac{1}{a} - \frac{1}{D} = \frac{1}{f}.$$

In fig. 44 the object is placed at a greater distance than the focal length, but the student can easily obtain the same result if it be taken between the principal focus and the lens.

Ex. 1. A small object is placed before a convex lens at a distance from it of twice the focal length: show that the image will be formed at the same distance on the other side of the lens, and that it will be of the same size as the object.

Here, $$\frac{1}{d} - \frac{1}{2f} = -\frac{1}{f}; \quad \therefore \quad d = -2f.$$

Also, since the object and image are at the same distance from the lens, their magnitudes must be equal (Art. 58).

Ex. 2. A candle-flame 2″ high is placed 10″ from a convex lens of focal length 6″: find the position and magnitude of the image.

$$\frac{1}{d}-\frac{1}{10}=-\frac{1}{6}; \quad \therefore \quad d=-15.$$

Hence the image is 15″ behind the lens.

$$\text{Also, } \frac{h}{h'}=\frac{D}{d} \text{ (Art. 58)}; \quad \therefore \frac{2}{h'}=\frac{10}{15}; \quad \therefore h'=3''; $$

hence the image is magnified to a height of 3″.

**Note.**—We have seen in the last example that if the distance of an object from a convex lens be equal to twice the focal length, there is a real image formed on the other side of the lens of the same size as the object. In the present example, the distance of the object from the lens is less than twice the focal length, but greater than the focal length, and we have obtained a real magnified image on the other side of the lens. The student may also see that, if the distance of the object from the lens be greater than twice the focal length, the real image formed will be smaller in size than the object.

Ex. 3. Two double convex lenses *A* and *B* have focal lengths 4 and 5 inches respectively, and are 10⅔ inches apart. Light diverges on *A* from a point 10 inches from it, and after passing through *A* falls on *B*. Find the position of the image formed by *B*.

Here, on refraction through *A*, we have

$$\frac{1}{d}-\frac{1}{10}=-\frac{1}{4}; \quad \therefore \quad d=-6\tfrac{2}{3}''.$$

Hence the conjugate focus of any point of the object with reference to the lens *A* is 6⅔ inches behind *A*, or 4 inches in front of *B*. The rays now diverge from this point and pass through *B*—

$$\frac{1}{d}-\frac{1}{4}=-\frac{1}{5};$$

therefore $d$ = 20″ in front of *B*, or 9⅓″ in front of *A*. This image is virtual and inverted.

## EXERCISES IX.

1. If an object be placed before a convex lens at a distance from it greater than the focal length, prove that there is an inverted real image formed behind the lens. Could this image be thrown on a screen?

2. If the distance of an object from a convex lens be less than the focal length, show that there is an enlarged virtual image formed. Could this image be thrown on a screen?

3. If $D$ be the distance of an object from a convex lens, and $f$ the focal length of the lens, prove that if $D$ is $> 2f$, the real image formed behind the lens is smaller in size than the object; whereas, if $D$ be $< 2f$ and $> f$, the real image formed behind the lens is larger than the object.

4. How may a convex lens be used to produce a virtual image of an object? Is this image magnified?

5. Show that a concave lens produces a virtual image of an object on the same side of the lens as the object is situated, and smaller in size.

6. A candle-flame is moved from a distant point on the axis of a convex lens up to the lens: describe the motion of the image.

7. An object 1″ high is placed 8″ from a convex lens of focal length 12″: find the position and magnitude of the image. Also describe the image.

8. A small object is placed 15″ from a concave lens of focal length 10″. Describe the image, and find its position.

9. A small object is placed 5″ in front of a convex lens of focal length 10″, and after passing through it falls on a concave lens of focal length 12·5″, situated at a distance of 15″ from the convex lens. Find the position of the image, and state if it is inverted.

10. Light diverges from a point $O$ on the axis of two convex lenses $A$ and $B$, and falls on the lens $A$, whose focal length is 6 cms., and whose distance from $O$ is 2 cms. It then falls on the lens $B$, whose focal length is 4 cms., and whose distance from $A$ is 15 cms. It then converges to a point $P$: find the position of $P$.

11. When an object is placed 13″ from a lens, the image is found to be 156″ from the lens on the opposite side. Find the focal length and the nature of the lens. Is this image real, and is it inverted?

12. Two double convex lenses $A$ and $B$, of focal lengths 2′ and 6″ respectively, are placed with their axes coincident. If the distance between them be 1′, find the position of the image, which the combination forms, of a small object on the axis 1′ 6″ from $A$ on the side remote from $B$.

13. A small luminous object is placed 7″ from a double convex lens of focal length 8″. After passing through, the light falls on another double convex lens of 6″ focal length, placed 4″ from the first lens. Find the position of the image. Is this image inverted?

# CHAPTER III.

## THE EYE AND OPTICAL INSTRUMENTS.

62. **Description of the Eye.**—The shape of the human eye is nearly that of a sphere, the curvature of which is greater in the anterior than in the posterior part. The outer coat *S* is hard and fibrous, and is called the *sclerotic* (σκλήρος, *hard*). The anterior portion *C* of the sclerotic is transparent, and is called the *cornea.* On account of its curvature being greater than that of the rest of the eye, it resembles a very convex watch-glass. The posterior portion of the sclerotic, commonly called the white of the eye, is opaque, and has a smooth and white appearance. Behind the cornea is an annular opaque diaphragm *I.* It constitutes the coloured part of the eye, and is called the *iris* (*iris, a rainbow*). The iris is pierced by a circular aperture *P*, called the *pupil.* By the action of involuntary muscles in the iris the aperture of the pupil is enlarged in partial darkness, so as to admit as much light as possible; whereas in very bright light the pupil is made to contract, so as to diminish the quantity of light entering the eye. Behind the pupil is the *crystalline lens L*,

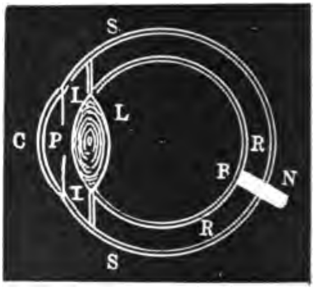

Fig. 46.

which is double convex, the convexity being greater behind than in front. It is not of uniform density throughout, being composed of layers or shells increasing in density inwards, the outermost shell at each curved surface having nearly the same index of refraction as the medium with which it is in contact—an admirable provision of nature which ensures that no light is lost by total reflection. The space between the crystalline lens and the cornea is called the *anterior chamber*, and is filled with a watery fluid called the *aqueous humour*. The larger space behind the crystalline lens is called the *posterior chamber*, and is filled with a transparent jelly-like substance called the *vitreous humour*.

63. The optic nerve $N$ enters at the posterior part of the eye, and expands into a fine nervous network $R$ on the back of the eye called the *retina* (*rēte, a net*), which, being sensitive to light, gives rise to the sensation of vision. The point $B$ where the optic nerve enters is not sensitive to light, and is called the *blind spot.*

As the index of refraction of the aqueous humour is 1·337, of the outer layer of the crystalline lens 1·337, of the central portion 1·4, and of the vitreous humour 1·339, it is evident that a pencil of rays entering the eye undergoes a series of refractions tending to converge the rays. A real inverted image is thus formed of any object within the range of vision, and if this image is formed on the retina, it can be seen distinctly.

64. **Power of Accommodation.**—By this term is meant the changes which can occur in the eye so as to enable it to see distinctly objects at various distances from it. Were the eye without this special power of adaptation, it would only be possible to see an object at a certain

particular distance from the eye. This would, in fact, be the case if the lens of the eye were fixed in shape and invariable in its focal length. In fact, the normal eye can, by the exercise of this power of accommodation, see any object distinctly whose distance varies from about 10 inches to infinity.

When the normal eye is at rest, parallel rays, *i.e.*, rays coming from infinity, are brought to a focus on the retina. Hence, in the case of normal eyesight, when looking at very distant objects where the rays may be regarded as parallel, the process of accommodation does not come into play; but, viewed from a closer distance, a sense of effort is felt in viewing the object which becomes greater with its nearer approach to the eye.

The changes in the lens to which the power of accommodation is due are principally effected by the action of the ciliary muscle which, by contracting, relaxes the suspensory ligaments which hold the crystalline lens in position, thereby causing the lens to become more convex, and, therefore, to bring rays to a focus on the retina which would otherwise meet behind the retina.

65. **Binocular Vision. Corresponding Points.**—Since each of our eyes forms an image on its retina, we should therefore expect that we would be conscious of two images of every object at which we look. This is certainly the case if we hold a finger between the two eyes, and quite close to them, when we see two fingers—one with the right and one with the left eye. However, on withdrawing the finger further from the eyes, the two images merge into one. The reason of this phenomenon is that to each point on the retina of one eye there is a definite *corresponding*

*point* similarly situated on the other, so that when an image of an object is formed at any point of the retina of one eye, and also an image at the corresponding point of the other retina, only one visual impression is produced, this being more intense than if viewed by one eye alone.

One great advantage of binocular vision is that we are able to form a judgment as to the distance of an object; for, in order to make the two images of an object fall on corresponding points, we have to converge the axes of the lenses of the eyes to a greater or less degree according as the object viewed is nearer or more remote.

Another advantage of binocular vision is that when we view a solid object, the two eyes see the object from different points of view; the image formed by the right eye is therefore different from that formed by the left, and hence, when these two images form a union, we get the perception of a solid standing out in relief—a result which could not be produced by one eye alone. This is the principle of the stereoscope, but it is not our purpose here to enter into any further details as regards the construction of this fascinating toy.

66. **Short Sight.**—We have seen that the normal eye can see objects distinctly whose distances vary from infinity to about 10″. In the case of *short sight* or *myopia*, the axis of the eye is too long, that is, the lens of the eye is too convex to suit its distance from the retina; consequently, parallel rays coming from infinity are, when the eye is at rest, brought to a focus in front of the retina, causing the object to be seen very indistinctly; hence, when the eye is at rest, a pencil of light must diverge from a point at a less distance than infinity in order to

be brought to a focus on the retina. Similarly, the least distance for distinct vision, when exercising the power of accommodation, is less than that of normal sight, that is, less than 10″. Hence, short-sighted persons cannot see distant objects clearly, but can have distinct vision of objects at a smaller distance from the eye than would be possible for a person with normal sight. Thus, if the distance for distinct vision ranged from a maximum of 6″ to a minimum of 3″, it would mean that for any distance beyond 6″ or less than 3″ there would be no distinct image formed. In order to correct this defect it is necessary to use *concave* spectacles, as concave lenses, by diverging the rays, enable images of distant objects to be formed on the retina instead of in front of it.

67. In fig. 47 is seen how a short-sighted person is unable to see clearly an object *AB*, placed at about 10″ from the eye, the image being at *A′B′* in front of the retina *R*. And in fig. 48 is seen how, by the aid of a concave lens *L*, the image is formed at *A′B′* on the retina *R*. In each case *O* is the centre of the lens, and the ray *AO* is not deviated in fig. 47, and only slightly deviated by the concave lens in fig. 48. The ray *AP*

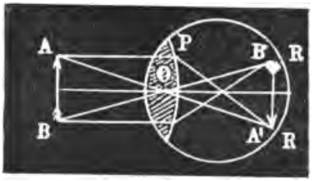

Fig. 47.

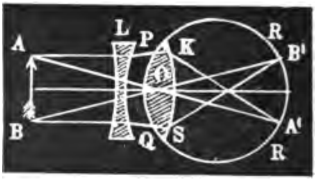

Fig. 48.

(fig. 48), on passing through the concave lens, is diverged along $PK$, and hence it meets with $AO$ at $A'$ on the retina, at a greater distance behind the lens of the eye than without the aid of the concave lens. Similarly $BQ$ is diverged along $QS$, and meets $BO$ on the retina at $B'$.

68. If the short-sighted person wishes to use concave glasses for reading purposes, to enable him to see an object such as a book clearly at the same distance as a person with normal sight would see it, the focal length of the spectacles required can be found thus:—

Let $d$ = the *greatest* distance at which the short-sighted person can see distinctly;

$D$ = the distance at which a normal person would hold the object, $D$ being greater than $d$.

Then, when the object is held at a distance $D$, the rays, after passing through the concave glasses, will appear to have come from a distance $d$. Hence

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f};$$

$$\therefore \quad f = \frac{Dd}{D-d}.$$

And as $D$ is greater than $d$, it follows that $f$ is positive, and hence the lens must be concave.

If, however, a short-sighted person requires glasses in order to see distant objects clearly, he must use lenses of a focal length different from the focal length of those by which he reads. What is now necessary is that parallel rays, *i.e.* rays coming from infinity, should enter the eye as if coming from a distance $d$, the greatest distance for distinct vision. Hence $D = \infty$, and we have

$$\frac{1}{d} - \frac{1}{\infty} = \frac{1}{f}; \quad \text{but } \frac{1}{\infty} = 0,$$

$$\therefore \quad \frac{1}{d} = \frac{1}{f}, \quad \text{or} \quad d = f.$$

That is, *for short-sighted persons, their long-distance glasses should have a focal length equal to the greatest distance for distinct vision.*

Ex. 1. The *greatest* distance for distinct vision of a short-sighted person is 6″. Find the focal length, and the kind of spectacles which he must use in order to see objects distinctly placed at the *least* distance for distinct vision in normal sight (10″).

Here, $D = 10''$, and $d = 6''$.

But
$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f};$$

$$\therefore \quad \frac{1}{6} - \frac{1}{10} = \frac{1}{f}, \quad \therefore f = 15'';$$

and, since $f$ is positive, the spectacles are concave.

Ex. 2. A student who, owing to continued use of the microscope, has become short-sighted, finds that his *greatest* distance for distinct vision is 8″. He wishes to provide himself with two pairs of spectacles—one for reading purposes, which will enable him to see print clearly at a distance of 10″, and the other to be used for looking at more distant objects. Find, in each case, the focal length of the concave glasses required.

Here, for reading purposes, $D = 10$, $d = 8$;

$$\therefore \quad \frac{1}{8} - \frac{1}{10} = \frac{1}{f}, \quad \therefore \quad f = 40''.$$

Again, in the case of long-distance glasses,

$$D = \infty, \quad \text{and} \quad d = 8;$$

$$\therefore \quad \frac{1}{8} = \frac{1}{f}; \quad \therefore \quad f = 8''.$$

69. **Long Sight.**—*Long Sight* or *hypermetropia* is the contrary of short sight. The eye, in this case, can see distant objects clearly, but cannot distinguish those which are close to the eye. The cause of this defect is that the axis of the eye is too short, that is, the lens of the eye is not convex enough to suit the length of the axis. Hence, when an object is held at what would be the least distance for distinct vision in a normal person, the lens of the eye, not being sufficiently convergent, will bring each pencil of rays to a focus behind the retina, and he finds that he must hold the object at a greater distance than 10 inches in order to see it clearly. The range for distinct vision is therefore from a point at a greater distance than 10 inches from the eye up to infinity, and beyond infinity to a point some distance behind the eye, so that rays entering the eye in a direction convergent to this point are brought to a focus on the retina.* To correct for this defect, convex lenses are used.

70. In fig. 49 is seen how a long-sighted person is unable to see clearly an object *AB*, placed at about 10″ from the eye, the image being at *A′B′* behind the retina *R*.

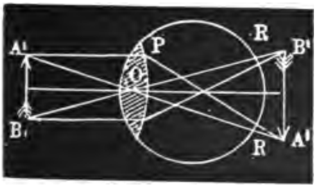

FIG. 49.

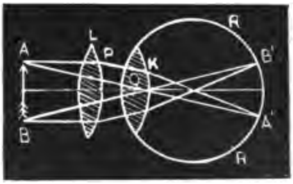

FIG. 50.

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* This is an example of the mathematical principle, that when the value of a variable quantity passes through infinity, it always changes its sign; hence the appearance of the point behind the retina.

And in fig. 50 is seen how, by the aid of a convex lens, *L* the image is formed at *A′B′* on the retina *R*. The ray *AP* (fig. 50), on passing through the convex lens *L*, is converged along *PK*, and hence it meets with *AO* at a less distance behind the lens of the eye than without the aid of the lens *L*.

71. The focal length of the glasses required by a long-sighted person can be determined, as in Art. 68, thus:—

Let $d$ = the *least* distance at which the long-sighted person can see distinctly;

$D$ = the corresponding distance for normal sight.

Then, when an object is held at a distance $D$, the rays, after passing through the convex glasses of the spectacles, will seem to have come from a distance $d$. Hence

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f};$$

$$\therefore \quad f = \frac{Dd}{D - d}.$$

Since $d$ is greater than $D$, the focal length $f$ is negative, and hence the lens is convex.

Ex. 1. The *least* distance of distinct vision of a long-sighted person is 25 inches. Find the focal length and the nature of the spectacles which he must use in order to see objects distinctly when placed at a distance of 10 inches from his eye.

Here, $D = 10, \quad d = 25;$

$$\therefore \quad \frac{1}{25} - \frac{1}{10} = \frac{1}{f}; \quad \therefore \quad f = -16\tfrac{2}{3}''.$$

Since the focal length is negative, the lenses are convex.

Ex. 2. In a case of long sight, rays which, before they enter the eye, are, when the eye is at rest, convergent to a point 3″ behind the lens of the eye (in this case the rays may be said to come from a distance $> \infty$ ) are brought to a focus on the retina: find what must be the focal length of glasses which, when the eye is at rest, converge parallel rays, *i.e.* rays coming from a distant object to a point on the retina.

Here the required lens is such that parallel rays passing through it will be converted into rays convergent to a point 3″ behind the lens. Hence $D = \infty$, and $d = -3$;

$$\frac{1}{-3} - \frac{1}{\infty} = \frac{1}{f}, \quad \text{or} \quad -\frac{1}{3} = \frac{1}{f}; \quad \therefore\ f = -3.$$

## EXERCISES X.

1. The *greatest* distance for distinct vision of a short-sighted person is 8″. Find the focal length and the nature of the spectacles he must use in order to see objects distinctly at a distance of 10″ from his eye.

2. A girl who has worked for long periods at fine needlework has become so short-sighted that her *greatest* distance of distinct vision is 5½″. She wishes to provide herself with two pairs of spectacles—one which will enable her to see clearly at a distance of 10″, and the other for looking at objects at long distances. Determine, in each case, the focal length of the concave glasses required.

3. If a person cannot see distinctly at a greater distance than 8″ from his eye, what kind of spectacles should he use, and what should be the focal length of the lenses in order that he may see clearly the print of a book held at a distance of 12″ from his eye?

4. The *greatest* and *least* distances for distinct vision of a short-sighted person are 8″ and 4″ respectively. If he makes use of *concave* spectacles whose focal length is 12″ to see more

distant objects, find the range of distances from his eyes within which he can place objects and still be able to see them distinctly when using the spectacles.

5. The *least* distance of distinct vision of a long-sighted person is 24″: find the focal length, and the nature of the glasses which will enable him to see clearly at 10″.

6. The *greatest* and *least* distances of distinct vision of a short-sighted person are 7″ and 4″ respectively: find within what limits the focal length of a pair of spectacles should lie in order to enable him to see objects distinctly at a distance of 10″ from his eyes.

7. The *nearest* point at which a person can see distinctly is 20″: find the focal length of the glasses which should be used in order that he may see distinctly at 12″.

72. **The Simple Microscope or Pocket-Lens.**—We have seen (Art. 59) that when an object is placed in front of a convex lens at a distance from it less than the focal length of the lens, a magnified virtual image of the object will be obtained. A convex lens thus used is called a simple microscope or magnifying glass.

73. **Magnifying Power of the Simple Microscope.**—The apparent magnitude or apparent diameter of a body is the angle it subtends at the observer's eye, and, in the case of microscopes, the magnification is the ratio of the apparent diameter of the image to that of the object, both being at the *least distance of distinct vision* of the observer.

Let $AB$ be the object (fig. 51) placed between $F$ and $O$ in such a position that a virtual image $A'B'$ is formed at the *least distance d for distinct vision* of the observer, whose eye is at $E$, the construction being the same as in Art. 59. Then $OQ' = d$, and $OQ = D$. Let the lines $AX$ and $BY$

parallel to the axis meet $A'B'$ in $X$ and $Y$ respectively. Then $AB = XY$, and the angle $A'OB'$ or $AOB$ is the angle at which he sees the object with the aid of the lens,

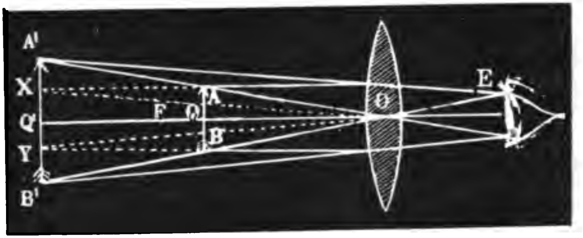

FIG. 51.

while $XOY$ is the angle at which he would see the object with the unaided eye if it were at the least distance $d$ of distinct vision. These angles being small, we have, expressing them in circular measure, the magnifying power

$$\begin{aligned} M &= AOB : XOY \\ &= \frac{AB}{D} : \frac{XY}{d} \\ &= \frac{AB}{D} : \frac{AB}{d}, \quad \text{for} \quad XY = AB \\ &= d : D. \end{aligned}$$

That is,
$$M = \frac{d}{D}.$$

But
$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f}; \quad \therefore \quad \frac{1}{D} = \frac{1}{d} - \frac{1}{f} = \frac{f-d}{df}.$$

Hence
$$\frac{d}{D} = \frac{f-d}{f},$$

$$\therefore \quad M = \frac{f-d}{f},$$

the value of $f$ being considered negative.

74. From this equation we can deduce a very interesting result, viz., that although a short-sighted person can, with the help of a convex lens, see the object at a greater angle than a long-sighted person, still the magnifying power of the lens for the former is less than for the latter. For, in the above formula, the value of $d$ for short sight is less than for long sight. But, since $f$ is negative in the above formula, the numerical value of the expression $f-d$ is really the sum of $f$ and $d$, and hence $f-d$, and therefore $M$, is less for a short-sighted than for a long-sighted person.

Ex. If the focal length of a convex lens be 2″, find the distance from it at which the object should be placed for a person whose least distance of distinct vision is 10″. Also find the magnifying power.

Here,
$$\frac{1}{10}-\frac{1}{D}=-\frac{1}{2}.$$

Hence
$$D = 1\tfrac{2}{3}''.$$

Also, the magnifying power

$$M=\frac{f-d}{f}=\frac{-2-10}{-2}=6 \text{ diameters.}$$

75. **The Compound Microscope.**—In its simplest form the compound microscope consists of two convex lenses, one $O$ with a short focal length, called the *object-glass* or *objective*, because it is close to the object; the other, $E$, whose focal length is considerably longer than that of $O$, is called the *eye-piece*, because it is close to the eye of the observer. The object $AB$ is placed just outside the principal focus $F$ of the object-glass $O$, and a real inverted enlarged image $A_1B_1$ is formed (Art. 57, also see note, p. 79). This image $A_1B_1$ is formed between the eye-piece $E$ and its principal focus $F_1$. Now the eye at $E$ views

this image in exactly the same way as a simple microscope views an object (Art. 73), and hence, by proper focussing, an enlarged virtual image $A_2B_2$ is formed at a distance $d$

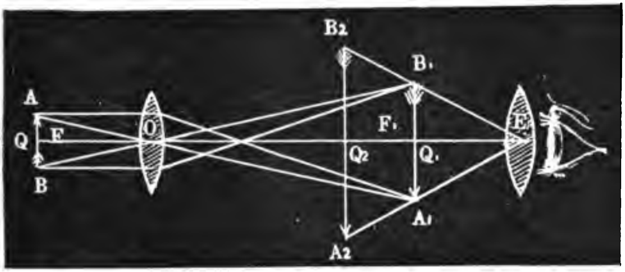

FIG. 52.

from $E$, where $d$ is the least distance of distinct vision of the observer. This image $A_2B_2$, being erect as regards the first image $A_1B_1$, is inverted with reference to the object $AB$.

*To find the Magnifying Power of a Compound Microscope.*

76. The magnifying power is the ratio of the angle $A_2EB_2$ to the angle which $AB$ would subtend at $E$ if it were at a distance $d$ from the eye equal to the least distance of distinct vision, *i.e.*, if $AB$ were at the same distance from $E$ as $A_2B_2$ is situated. Hence, substituting the circular measures of these angles, we have the magnifying power

$$M = \frac{A_2B_2}{d} \div \frac{AB}{d} = \frac{A_2B_2}{AB} = \frac{A_2B_2}{A_1B_1} \cdot \frac{A_1B_1}{AB}.$$

But $$\frac{A_2B_2}{A_1B_1} = \frac{d}{D},$$

where $d$ = distance of distinct vision,
and $D$ = distance $EQ_1$.

Also,
$$\frac{A_1B_1}{AB} = -\frac{d_1}{\Delta},$$

where $d_1 = OQ_1$ (regarded as negative),

and $\Delta$ = distance $OQ$ of object from object-glass.

$$\therefore \quad M = -\frac{d}{D} \times \frac{d_1}{\Delta}.$$

But since $\frac{1}{d} - \frac{1}{D} = \frac{1}{f}$, where $f$ = focal length of $E$,

$$\therefore \quad \frac{d}{D} = \frac{f-d}{f}, \quad \text{as in Art. 73.}$$

And since $\frac{1}{d_1} - \frac{1}{\Delta} = \frac{1}{f_1}$, where $f_1$ = focal length of $O$,

$$\therefore \quad \frac{d_1}{\Delta} = \frac{f_1}{f_1 + \Delta}.$$

Hence
$$M = -\frac{f-d}{f} \times \frac{f_1}{f_1 + \Delta},$$

where $f$ = focal length of eye-piece,
$f_1$ = focal length of object-glass,
$d$ = distance of distinct vision,
$\Delta$ = distance of object from object-glass.

In applying the above formula to practical examples, it is to be borne in mind that $f$ and $f_1$, being the focal lengths of convex lenses, are negative quantities. The length $OE$ of the tube of the microscope when the image is seen is equal to the sum of $OQ_1$ and $EQ_1$, that is, $d_1 + D$.

77. From the above formula follows the same apparent paradox, as in the case of the simple magnifying glass (Art. 74), that, although a short-sighted person, with the aid of a microscope, sees an object at a greater angle than

a long-sighted person, still the magnifying power of the instrument is less for the former than for the latter. This follows from the above formula, as in Art. 74.

Ex. 1. The focal length of the object-glass of a microscope is $\frac{1}{4}''$, and of the eye-piece $\frac{4}{5}''$, and the object is placed $\cdot 255''$ from the object-glass. Supposing the distance for distinct vision of the observer to be $10''$, find the magnifying power and the length of the microscope.

Here, $f = -\cdot 8$, $f_1 = -\cdot 25$, $d = 10$, and $\Delta = \cdot 255$, and

$$M = -\frac{f-d}{f} \times \frac{f_1}{f_1 + \Delta} = \frac{\cdot 8 + 10}{\cdot 8} \times \frac{\cdot 25}{\cdot 005} = 675 \text{ diameters.}$$

Also, the length of the tube

$$= OQ_1 + EQ_1 \text{ (fig. 52)} = d_1 + D.$$

But $$\frac{1}{d_1} - \frac{1}{\cdot 255} = -\frac{1}{\cdot 25}; \quad \therefore \quad d_1 = -12\cdot 75''.$$

Also, $$\frac{1}{10} - \frac{1}{D} = -\frac{1}{\cdot 8}; \quad \therefore \quad D = \cdot 74''.$$

Hence, since length of tube $= d_1 + D$ irrespective of sign,

$$\text{required length} = 12\cdot 75 + \cdot 74 = 13\cdot 49''.$$

Ex. 2. In the last example, if the distance of distinct vision be $6''$, find the magnifying power.

Here, $$M = \frac{\cdot 8 + 6}{\cdot 8} \times \frac{\cdot 25}{\cdot 005} = 425 \text{ diameters.}$$

Hence the magnifying power is less for a short-sighted than for a long-sighted person—a result which is explained in Arts. 74, 77.

**78. The Astronomical Telescope.**—This instrument is similar to the compound microscope, and, like it, consists of a convex object-glass and a convex eye-piece (fig. 53). It is used for observing the heavenly bodies, and therefore the object $AB$ is at a very great distance from the object-glass, instead of being, as in the microscope, just outside the principal focus. Hence the rays from the object $AB$, being almost parallel, the inverted image $A_1B_1$, which is first formed, is almost at the principal focus of the object-glass. The eye-piece $E$ is of very small focal length, and, as in the case of the microscope, a magnified virtual image $A_2B_2$ is formed at a distance from $E$ equal to the least distance of distinct vision.

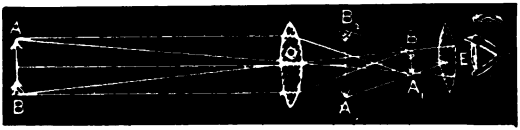

FIG. 53.

**Note.**—In the case of telescopes, the *magnifying power* is the ratio of the angle subtended at the eye by the image to the angle which the object at its actual distance subtends at the eye.

### *Magnifying Power of Astronomical Telescope.*

79. Let $f$ = the focal length of the eye-piece,
and $f_1$ = the focal length of the object-glass.

The angle subtended by $A_2B_2$ at $E = A_2EB_2 = A_1EB_1$, and the angle subtended by $AB$ at $E$ is, on account of the great distance of $AB$, the same as that subtended at $O = AOB = A_1OB_1$.

Hence $$M = \frac{A_1EB_1}{A_1OB_1};$$

therefore, on substituting the circular measures of these angles, we have, remembering that the distance of $A_1B_1$ from $O$ is the focal length $f_1$ of the object-glass,

$$M = \frac{A_1B_1}{D} \div \frac{A_1B_1}{f_1} = \frac{f_1}{D},$$

regardless of sign, where $D$ is the distance of $A_1B_1$ from $E$.

But $$\frac{1}{d} - \frac{1}{D} = \frac{1}{f},$$

where $d$ is the distance of $A_2B_2$ from $E$, that is, the distance of distinct vision.

$$\therefore \quad D = \frac{fd}{f-d}.$$

Hence $$M = \frac{f_1}{D} = \frac{f_1}{f} \times \frac{f-d}{d},$$

or, giving $M$ a positive value,

$$M = \frac{f_1}{f}\left(1 - \frac{f}{d}\right).$$

80. From this formula it follows that, in the use of the astronomical telescope, the magnifying power is greater for short-sighted than for long-sighted persons; for, owing to the fact that $f$ is negative, the expression

$1 - \dfrac{f}{d}$ is numerically equal to the sum of 1 and $\dfrac{f}{d}$,

which is greater for a short-sighted than for a long-sighted person, owing to the fact that the value of $d$ for the former is less than for the latter.

The above result is considerably simplified when the observer's eye is capable of bringing parallel rays to a focus on the retina. In this case, the image $A_1B_1$ can be viewed at a distance from $E$ equal to $f$, the focal length of $E$; and, since its distance from $O$ is also equal to the focal length $f'$ of $O$, the rays from $A_1B_1$ then enter the eye as parallel rays.

Hence $$M = \frac{f_1}{f},$$

a result which can also be deduced from above formula by making $d = \infty$. That is, when the observer's eye is capable of bringing parallel rays to a focus on the retina, *the magnifying power is the ratio of the focal lengths of the lenses, and the length of the tube is the sum of their focal lengths.*

Ex. 1. Find the magnifying power of an astronomical telescope, the focal length of whose object-glass is 12′, and of the eye-glass $\frac{1}{2}''$, for a person whose distance for distinct vision is—(1) 6″, (2) 18″.

(1) Here, $f = -\frac{1}{2}''$, $f_1 = -144''$, $d = 6''$.

$$\therefore \quad M = \frac{-144}{-\frac{1}{2}}\left(1 + \frac{\frac{1}{2}}{6}\right) = 288 \times \tfrac{13}{12} = 312 \text{ diameters;}$$

$$(2) \quad M = 288\left(1 + \frac{\frac{1}{2}}{18}\right) = 288 \times \tfrac{37}{36} = 296 \text{ diameters,}$$

the magnifying power, as was explained in Art. 80, being greater for the short-sighted than for the long-sighted person.

Ex. 2. In the last example, find the magnifying power for a person whose eye can bring parallel rays to a focus on the retina. Also, find the distance between the two glasses.

Here, $$M = \frac{f_1}{f} = \frac{144}{\frac{1}{2}} = 288 \text{ diameters.}$$

Also, the distance between the two glasses is the sum of their focal lengths = $144\frac{1}{2}''$.

81. **Galileo's Telescope.**—This telescope consists of two lenses, namely, a convex object-glass $O$ (fig. 54), and a concave eye-piece $E$. The rays from the distant object $AB$ would, if there were no concave eye-piece to intercept them, form an inverted real image $A_1B_1$ at the principal focus of the object-glass; but the concave eye-piece is interposed at a distance from $A_1B_1$ greater than its own focal length. Then the rays from $A$, which would otherwise meet at $A_1$, are diverged on passing through the concave eye-piece, and enter the eye as if coming from $A_2$. Similarly, the rays from $B$ are diverged as if coming from $B_2$, and a magnified virtual image, which is not inverted, is formed at $A_2B_2$ at a distance from $E$ equal to $d$, the least distance of distinct vision of the observer.

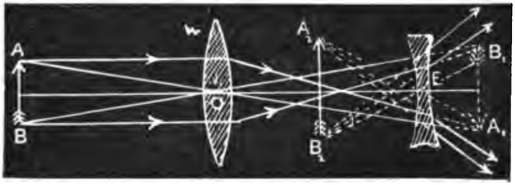

FIG. 54.

82. *Magnifying Power.*—As before, let $f$ and $f_1$ be the focal lengths respectively of the eye-piece and object-glass.

Now, as in the case of the astronomical telescope, the magnifying power is the ratio of the angles subtended by $A_2B_2$ and $AB$ at $E$, and the angle subtended by $AB$ at $E$ is almost the same as that subtended at $O$. Hence

$$M = \frac{A_2EB_2}{AOB} = \frac{A_1EB_1}{A_1OB_1}.$$

On substituting the circular measures of these angles, we have

$$M = \frac{A_1B_1}{D} \div \frac{A_1B_1}{f_1} = \frac{f_1}{D},$$

where $D$ is the distance of $A_1B_1$ from $E$.

But $$\frac{1}{d} - \frac{1}{-D} = \frac{1}{f}, \quad D \text{ being negative};$$

$$\therefore \quad D = \frac{fd}{d-f}.$$

But $M = -\frac{f_1}{D}$, regard being had to the sign of $f_1$;

$$\therefore \quad M = -\frac{f_1}{f} \times \frac{d-f}{d} = -\frac{f_1}{f}\left(1 - \frac{f}{d}\right).$$

83. This result, as in the case of the astronomical telescope, is greatly simplified when the vision is normal, and is therefore capable of bringing parallel rays to a focus on the retina. In this case, the first image $A_1B_1$ can be at a distance behind $E$ equal to its focal length, and hence $D = f$, and the magnifying power is given by the equation

$$M = -\frac{f_1}{f},$$

a result which might also be deduced from the foregoing formula by making $d = \infty$.

Hence, in the Galilean telescope, when the eye is able to focus parallel rays on the retina, *the distance between the glasses is the difference between their focal lengths, and the magnifying power is the ratio of their focal lengths.*

84. The Galilean telescope possesses the following advantages :—

1. It gives images in their right position with the aid of only two lenses.

2. As it has only two lenses, it absorbs very little light.

3. For normal vision the distance between the two lenses is the difference of their focal lengths, and not their sum, as in the case of the astronomical telescope. It is therefore short and portable.

The following are its disadvantages :—

1. On account of the divergence of the rays, its field of view is very small.

2. It does not admit of the employment of cross-wires, as is the case with the astronomical telescope; for, in order that cross-wires may be used, they must be made to coincide with a real image which does not exist in Galileo's telescope.

85. The *opera-glass* consists of a pair of Galilean telescopes, with their axes parallel and so constructed that by a screw arrangement the distance between the lenses of each can be altered so as to obtain clear images of objects at various distances from the observer.

Ex. 1. In an opera-glass the focal length of each object-glass is 8″, and of each eye-piece 2″. Find the magnifying power for a person whose distance of distinct vision is 10″.

Here, $$M = -\frac{f_1}{f}\left(1 - \frac{f}{d}\right) = -\frac{-8}{2}\left(1 - \tfrac{2}{10}\right) = 3{\cdot}2 \text{ diameters.}$$

Ex. 2. In Example 1, what is the magnifying power to a person who can bring parallel rays to a focus on the retina, and what is the distance between the two lenses?

Here, $$M = -\frac{f_1}{f} = -\frac{-8}{2} = 4 \text{ diameters.}$$

The distance between the glasses is the difference between the focal lengths $f_1$ and $f$, regardless of sign, and is, therefore 6″.

86. **Terrestrial Telescope.**—In an astronomical telescope it is no inconvenience that the object viewed is inverted, but it would be a decided drawback in a telescope used for terrestrial purposes. In order to produce an erect image in this instrument, two additional convex lenses whose centres are $O_1$ and $O_2$ are introduced (fig. 55). The lens $O_1$ is so placed that the real inverted image $A_1B_1$, first formed by the object-glass, is at its principal focus. The rays passing through $A_1$ will therefore emerge from the lens $O_1$ parallel to the secondary axis through $A_1$, viz., $A_1O_1$. Similarly, the rays through $B_1$ will emerge from the lens $O_1$ parallel to $B_1O_1$. Now, the rays which fall upon the lens $O_2$ being parallel, an erect image $A_2B_2$ is formed at the principal focus of $O_2$. This image is viewed through the eye-piece $E$, and an enlarged virtual image $A_3B_3$ is formed at the distance of distinct vision of the observer.

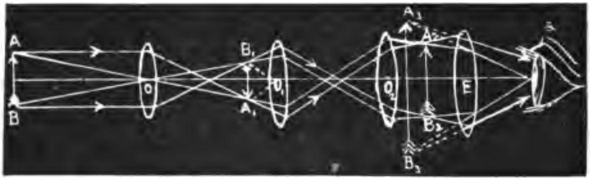

FIG. 55.

## EXERCISES XI.

1. If the focal length of a convex lens be 2″, find the distance at which an object should be placed by a person whose distance of distinct vision is 6″. Find also the magnifying power.

2. In Example 1, if the distance of distinct vision be 8″, find where the object should be placed, and find the magnifying power.

**Note.**—The student will notice that the last two examples illustrate the fact that the magnifying power is greater as the distance of distinct vision increases.

3. In a microscope, if the focal length of the object-glass be ·2″, and of the eye-glass ·8″, the object being placed at a distance from the object-glass of ·203″, find the magnifying power for a person whose distance of distinct vision is 10″. Find also the length of the microscope.

4. In Example 3, if the distance of distinct vision be 6″, find the magnifying power.

5. Find the magnifying power of a simple refracting astronomical telescope whose object-glass has a focal length of 5′ and eye-piece ½″, the distance of distinct vision being 10″.

6. In Example 5, find the magnifying power and the length of the tube, when used by a person whose eye has the power of bringing parallel rays to a focus on the retina.

7. Find the magnifying power of a simple refracting astronomical telescope whose object-glass is of 4′ and eye-glass ⅓″ focal lengths, where the distance of distinct vision is—(1) 10″, (2) 5″.

8. In an opera-glass the focal length of the object-glass is 9″, and of the eye-piece 2″, while the distance of distinct vision is 6″: find the magnifying power.

9. In Example 8, if the eye of the observer can bring parallel rays to a focus on the retina, find the magnifying power and the length of the tube.

10. Explain how the magnifying power of a Galilean telescope is greater for a long-sighted than for a short-sighted person.

87. **Reflecting Telescopes.**—The telescopes which have already been described are refracting instruments. In reflecting telescopes, the object-glass is replaced by a concave mirror, called a *speculum*, composed of highly polished metal.

**Herschel's Telescope.**—In this telescope the concave speculum $M$ is so inclined that the inverted image $A_1B_1$ of the heavenly body is formed near the side of the telescope. This image is viewed by the observer through the eye-piece $E$ in the same way as in a refracting telescope, and a magnified image $A_2B_2$ is produced. Herschel's telescope was constructed in 1789; its length was 40′, and the diameter of the speculum 50″, and its magnifying power exceeded anything previously attempted.

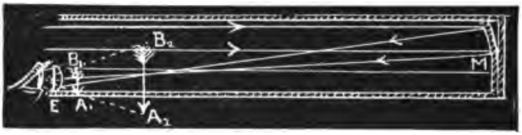

FIG. 56.

The magnifying power of Herschel's telescope is evidently obtained by the same formula as for the simple refracting telescope, for the concave mirror merely replaces the convex object-glass.

Hence $$M = -\frac{f_1}{f}\left(1 - \frac{f}{d}\right),$$

where $f$ = the focal length of the eye-piece,
$f_1$ = the focal length of the concave mirror,
and $d$ = distance of distinct vision.

Also, where the eye is capable of bringing parallel rays to a focus on the retina, the magnifying power

$$M = -\frac{f_1}{f}.$$

88. **The Newtonian Telescope.**—This instrument consists of a concave speculum $M$ at one end of the tube, the open end of which, as in Herschel's, is directed towards the object. The rays from the object are reflected from the concave speculum, and would form an image of the object at its principal focus, only the rays before they meet are intercepted by a plane mirror $n$, set at an angle of 45° to the axis of the telescope; they are then reflected,

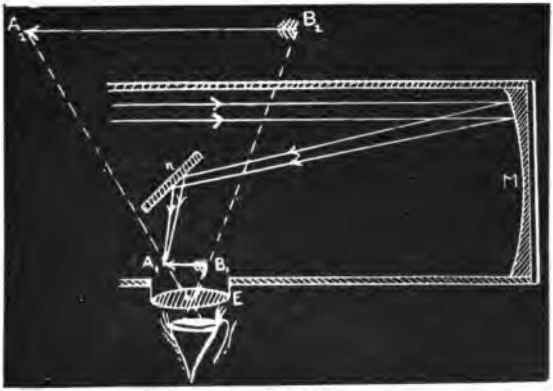

FIG. 57.

forming an image $A_1B_1$. This image $A_1B_1$ is viewed by an eye-piece $E$ in the side of the tube, and a magnified image $A_2B_2$ is produced.

The plane mirror has no influence on the magnifying power, which is therefore found in the same way as in Herschel's telescope. In fig. 55, only the rays from the object which form the image $A_1$ are shown, but the formation of the image $B_1$ can be similarly shown.

The great reflecting telescope, constructed by the Earl of Rosse in 1845 at Birr Castle, Ireland, far exceeded in power any previous instrument: the diameter of the speculum is 6′, and the focal distance 53′. It is at present being used as a Newtonian telescope, although it can be altered so as to view the image from the front, as in Herschel's telescope.

89. **The Gregorian Telescope.**—This telescope takes its name from James Gregory of Aberdeen, the inventor of reflecting telescopes. His telescope was planned in 1663, but not constructed until a later date, while Newton's was actually constructed in 1666.

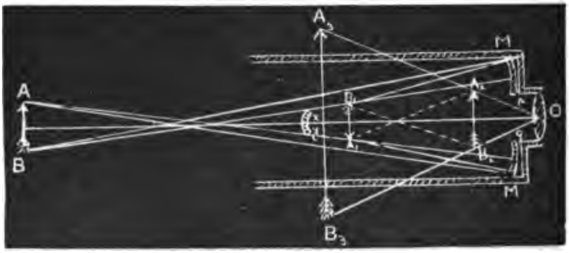

FIG. 58.

The Gregorian telescope consists of a concave speculum *MM*, pierced through the centre by an aperture *pq*, through which the observer looks. The object $AB$ is supposed to be very far away, and rays from $A$ striking the speculum mirror are reflected to $A_1$, almost at the principal focus of $M$. Similarly, rays from $B$ are reflected to $B_1$, and hence there is a small inverted image at $A_1B_1$. Behind this image $A_1B_1$ is a very small concave mirror $xy$, at a distance from $A_1B_1$ greater than its focal length, and hence an enlarged re-inverted image $A_2B_2$ is formed by this mirror $xy$. This image $A_2B_2$ is viewed by the observer's eye through a convex lens placed at $O$, $A_2B_2$ being just outside the principal focus of $O$, and an enlarged erect image $A_3B_3$ is formed.

90. **Cassegrain's Telescope.**—This instrument was invented in France by Cassegrain in 1672. It consists, like Gregory's, of a concave speculum, with a hole through the centre. The rays from the object are reflected from the speculum, but, instead of being allowed to meet as in Gregory's, they are intercepted by a small *convex* mirror. These rays being convergent to a point just behind the convex mirror, an image is formed just in front of the aperture in which the eye-piece is placed, and this image is viewed through the eye-piece as before.

## EXERCISES XII.

1. In the Gregorian telescope, if $f$, $f_1$, and $f_2$ denote the focal lengths of the eye-glass, the speculum, and the small mirror respectively: prove that the magnifying power $M$, for a person whose eye can bring parallel rays to a focus, is given by the equation

$$M = \frac{f_1 + f_2}{f_2} \times \frac{f_1}{f}, \quad \text{approximately.}$$

2. In Cassegrain's telescope, using the same notation as in Example 1, prove

$$M = \frac{f_1 - f_2}{f_2} \times \frac{f_1}{f}, \text{ approximately.}$$

3. Find the magnifying power of Herschel's telescope, having a speculum of 40 feet focal length, the focal length of the eye-piece being ½ inch, and the distance of distinct vision 6 inches.

4. Find the magnifying power of a Gregorian telescope for a person whose eye can bring parallel rays to a focus on the retina, the focal length of whose speculum is 6 feet, that of the small mirror 3 inches, and that of the eye-piece ½ inch. (See Question 1.)

5. Given the same quantities for a Cassegrain telescope as in Question 4, find the magnifying power. (See Question 2.)

91. **Magic Lantern.**—The principle of this instrument is as follows:—A bright lamp or limelight is placed at *F*, the principal focus of a concave mirror *M*. The light is therefore reflected from *M* as parallel rays. These rays, falling on the convex lens *L*, are concentrated on the picture *AB* painted on the slide *S*. There is a convex lens *C* at a distance from the picture *AB* slightly greater than its focal length; hence a real magnified inverted image $A_1B_1$ is thrown on the screen (Art. 57). Evidently the magnifying power is the ratio of $RC : CS$. The tube containing the lens *C* can be moved inwards or outwards so as to focus the image as clearly as possible on the screen.

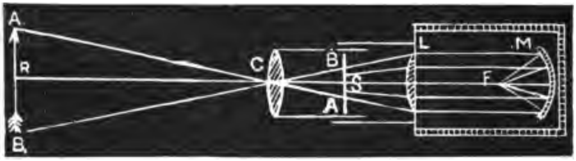

Fig. 59.

*Dissolving views* are produced by using two lanterns similar to one another, so that a picture from each can be focussed on the same part of the screen. The object-glass of each can be covered with shades, which are worked in such a manner that when one shade is being raised the other is being lowered, and therefore one picture gradually disappears while the other comes into view.

92. **Camera Obscura.**—This dark chamber is a kind of tent surrounded by dark curtains. At the top is a revolving box, containing a plane mirror *M*, inclined at an angle of 45°. The rays from an external object *AB* are reflected from the plane mirror *M*, and pass through a convex lens *L*, when a real image *A′B′* is formed on a white horizontal screen. The box containing the mirror and lens can be rotated round so as to form successive images of objects on all sides of the tent.

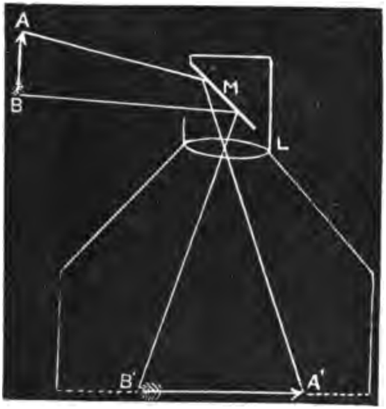

FIG. 60.

# CHAPTER IV.

## DISPERSION OF LIGHT.

93. **Decomposition of White Light.**—*Spectrum.*—When a ray of white light, as, for example, a ray of sunlight, passes from one medium into another of different refractive index, it is decomposed into several kinds of rays of different colours. This decomposition of the ray is called *dispersion*.

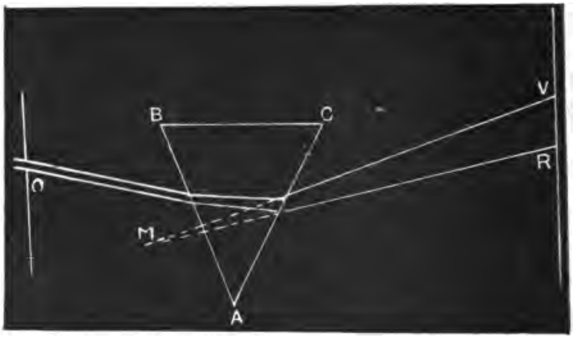

Fig. 61.

94. **Newton's Experiment.**—Let a beam of light from the sun be admitted into a dark room through a small aperture *O*. The pencil which enters the room from *O* would form a rounded white image of the sun if allowed to fall directly on a screen. But if a glass prism *ABC* be interposed, with its refracting edge *A* horizontal,

the beam is refracted away from the angle $A$, and when allowed to fall on the screen produces an elongated vertical image $VR$, rounded at the ends, and composed of horizontal bands of different colours. This image that is now seen is actually made up of numbers of overlapping images of different colours, each similar in shape to the white image that would have been formed were the prism not interposed. This band, coloured like a rainbow, is called the *solar spectrum*.

In this solar spectrum there are really an innumerable number of shades of colour, varying from violet, which is the most refrangible, to red, which is least. It is usual, however, to distinguish seven principal primary colours—namely, *violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, and *red*; the order here given being the same as is observed in the spectrum.

From Newton's experiment it is evident that a beam of white light is not homogeneous, but is made up of an infinite variety of rays, ranging from violet to red. When passed through the prism (fig. 61), the violet rays are bent most, and appear at $V$, while the red rays are bent least, and are seen at $R$. Newton found that the refractive indices of violet and red rays, when passing from air into crown glass, are $\frac{78}{50}$ and $\frac{77}{50}$ respectively, the refractive indices of the other rays having intermediate values.

95. **Dispersion. Definition.**—The angle between the directions of the refracted violet and red rays is called the *dispersion*.

Thus the angle at $M$ (fig. 61) is the dispersion; and, since the rays before entering the prism were in the same or parallel direction, it is evident that *the dispersion is measured by the difference between the deviations of the violet and red rays.*

Hence, if $\delta_v$ and $\delta_r$ represent the deviations of violet and red rays respectively, and $\mu_v$ and $\mu_r$ their respective refractive indices for a given substance, the dispersion produced by a thin prism, whose refracting angle is $\theta$, can thus be found. We know

$$\delta_v = (\mu_v - 1)\theta \quad \text{(Art. 42)},$$

and

$$\delta_r = (\mu_r - 1)\theta;$$

subtract,

$$\therefore\ \delta_v - \delta_r = (\mu_v - \mu_r)\theta.$$

Then, if $\Delta$ represent the dispersion, we have

$$\Delta = (\mu_v - \mu_r)\theta.$$

96. **Dispersive Power. Definition.**—The *dispersive power* of a substance is the ratio of the dispersion of a thin prism of the substance to the mean deviation of the violet and red rays.

The mean index of refraction of violet and red rays is $\frac{1}{2}(\mu_v + \mu_r)$. Let this mean index of refraction be denoted by $\mu$, and the dispersive power of the prism by $\phi$, and we have, from the above definition,

$$\text{dispersive power} = \frac{\text{dispersion}}{\text{mean deviation}},$$

or

$$\phi = \frac{(\mu_v - \mu_r)\theta}{(\mu - 1)\theta},$$

or

$$\phi = \frac{\mu_v - \mu_r}{\mu - 1}.$$

Ex. 1. If the refractive indices of the red and violet rays for fluor-spar be 1·429 and 1·439 respectively, find the dispersion for a prism of this substance whose angle is 4°.

Here $\Delta = (\mu_v - \mu_r)\theta$

$$= (1{\cdot}439 - 1{\cdot}429)\,4^\circ = {\cdot}01 \times 240' = 2'\,24''.$$

Ex. 2. If the refractive indices of the extreme violet and red rays for crown glass be 1·56 and 1·54 respectively, find its dispersive power.

Here $\mu$, the mean index of refraction

$$= \tfrac{1}{2}(1{\cdot}56 + 1{\cdot}54) = 1{\cdot}55.$$

Hence $$\phi = \frac{\mu_v - \mu_r}{\mu - 1} = \frac{{\cdot}02}{{\cdot}55} = {\cdot}036.$$

97. **Dispersion of a Lens. Chromatic Aberration.**—When a pencil of white light of which one ray is *AB* (fig. 62) passes through a lens, the rays undergo dispersion which, in the case of the ray *AB*, would be the same as if it passed through the prism *PMN*, formed by tangent planes at *B* and *C*, the points at which the ray enters and leaves the lens.

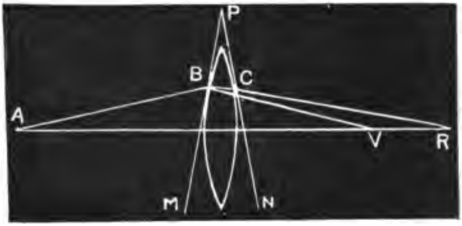

FIG. 62.

The violet rays, being refracted through the largest angle, will be brought to a focus at *V*, while the red rays, being least refrangible, converge to a focus at *R*.

The indigo, blue, green, yellow, and orange rays converge to points between $V$ and $R$. Hence the light is not brought to one focus, the rays of each colour having a separate focus; and hence, if an image of the luminous body from which the rays proceed be thrown on a screen placed near $V$ or $R$, this image will be coloured at its edges, the distance $VR$ being called the *chromatic aberration of the lens.*

To calculate the dispersion of a lens, we have (Art. 96)

$$\phi = \frac{\Delta}{\delta}, \quad \text{or} \quad \Delta = \phi\delta.$$

But it can easily be shown that $\delta = \frac{a}{f}$, where $a$ is the distance of $B$ or $C$ from the axis (this distance is called the *semi-aperture*). For, if $AB$ be taken parallel to the

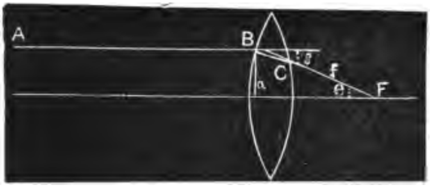

FIG. 63.

axis (fig. 63), it is refracted to the principal focus $F$, the angle $\delta$ being the deviation. But $\delta = \theta$, and $\theta = \frac{a}{f}$ expressed in circular measure;

$$\therefore \quad \delta = \frac{a}{f}. \qquad \text{Hence} \quad \Delta = \phi\frac{a}{f}.$$

That is, *the dispersion of a lens is directly proportional to the dispersive power and the aperture, and inversely proportional to the focal length.*

98. **Achromatic Lenses. Newton's Mistake.**—We have seen (Art. 97) that a single lens of homogeneous material will not form a single white image of a white object, but a series of coloured images, the focal length of the lens being different for each of the coloured rays. In order to discover some method of remedying this evil, Newton experimented with a compound prism, composed of one of glass and the other of water, with their refracting angles turned in the opposite directions; and he found that, when rays of white light were passed through, it was only when the rays emerged parallel to their original directions that no decomposition into colours occurred; that is, it was only when there was no deviation that there was no dispersion. Hence he concluded that the construction of an *achromatic prism*, and hence of an *achromatic lens*, was impossible. He also concluded that the dispersion is proportional to the mean deviation, and hence that the dispersive powers of all substances are the same, for

$$\text{dispersive power} = \frac{\text{dispersion}}{\text{mean deviation}}.$$

Newton's mistake arose from the fact that, by a mere accident, the glass prism he made use of had the same dispersive power as water, and, had he repeated his experiment with another pair of substances, he would have discovered his error.

Many years afterwards Dolland, a London optician, discovered Newton's error, and was thus enabled to manufacture achromatic instruments.

99. **Definition.**—Two lenses are said to *achromatise* when their dispersions are equal and opposite.

If $\Delta$ and $\Delta'$ be the dispersions of two such lenses, and $\phi$ and $\phi'$ their respective dispersive powers, the semi-aperture $a$ being the same in each case, then

$$\Delta = \frac{\phi a}{f} \quad \text{and} \quad \Delta' = \frac{\phi' a}{f'};$$

$$\therefore \quad \frac{\phi a}{f} = -\frac{\phi' a}{f'}, \quad \text{or} \quad \frac{\phi}{f} = -\frac{\phi'}{f'}, \quad \text{or} \quad \frac{\phi}{\phi'} = -\frac{f}{f'}.$$

From this we conclude that if one lens is concave and the other convex, they will achromatise, provided their *dispersive powers are proportional to their focal lengths.*

Usually one lens *A* is a diverging convexo-concave lens of flint glass; the other *B* is a double convex lens of crown glass, with focal lengths proportional to the dispersive powers of flint and crown glass. A ray of white light on passing through these lenses will therefore emerge as white light, or, in other words, there is no chromatic aberration.

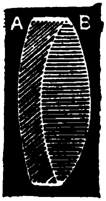

FIG. 64.

Ex. Find the dispersion produced by a convex lens of crown glass whose semi-aperture is 1 inch, and whose focal length is 2 feet, the dispersive power of crown-glass being ·036.

Here, $\Delta = \phi \frac{a}{f} = \cdot 036 \times \frac{1}{24}$

$= \cdot 0015$ (expressed in circular measure)

$= \cdot 0015 \times 206265''$

$= 5'\ 9{\cdot}4''$.

## EXERCISES XIII.

1. If the refractive indices of the red and violet rays for crown glass be, as given by Newton, $\frac{77}{50}$ and $\frac{78}{50}$, find the dispersion for a prism whose angle is 2°.

2. Taking the refractive indices of red and violet rays for crown glass to be, as given by Newton, viz., $\frac{77}{50}$ and $\frac{78}{50}$, find the dispersive power of crown glass.

3. A ray of light is incident, nearly perpendicularly, upon a prism of rock-salt whose angle is 2°: find the dispersion of the ray, given that $\mu$ for rock-salt is 1·557, and that the dispersive power of rock-salt is ·053.

Here, since
$$\phi = \frac{\Delta}{\delta}, \quad \therefore \Delta = \phi\delta;$$
or
$$\begin{aligned} \Delta &= \phi(\mu - 1)\theta \\ &= {\cdot}053\,(1{\cdot}557 - 1)\,2^\circ \\ &= 3'\,23''. \end{aligned}$$

4. Find the dispersion produced by a water prism whose angle is 3°, the dispersive power of water being ·035.

5. Find the dispersion produced by a lens of fluor-spar whose semi-aperture is 2″, and whose focal length is 10″, the dispersive power of fluor-spar being ·022.

6. Show that a convex lens of crown glass of semi-aperture 1″ and 3′ focal length, and a concave lens of flint glass of the same aperture and of focal length 4′, when combined, will form an achromatic combination, *i.e.* a ray of white light on passing through these lenses will emerge as white light, the dispersive powers of crown and flint glass being ·036 and ·048 respectively. Also find the dispersion in each case.

7. Find the focal length of a water-lens which will achromatise a lens of fluor-spar, the dispersive powers of water and fluor-spar being ·035 and ·022 respectively.

# ANSWERS.

## I. (Pages 17, 18.)

1. First mirror, 1′, 5′, 7′; second mirror, 2′, 4′, 8′.

3. Its height and breadth should be one-half the height and one-half the greatest breadth of the spectator respectively.

4. The length of each side of the mirror is one-third that of the wall.

5. One mile. 6. 19. 7. (1) 4; (2) 5. 10. (1) 45°; (2) 60°.

## II. (Page 27.)

1. $4\frac{2}{7}''$ behind the mirror. 2. 3·68″ in front of the mirror.

3. 2·73″ behind. 4. 20″ behind. 5. (1) 6′; (2) − 6″.

6. Mirror is concave; $r = 9{\cdot}23''$; $f = 4{\cdot}61''$.

7. Mirror is convex; $r = -17{\cdot}14''$; $f = -8{\cdot}57''$.

## III. (Pages 34, 35.)

6. 36″ in front; 2″ long; real.

7. 3″ behind; diameter, $\frac{3}{4}$ inch; virtual.

8. 2′ from the screen; $4\frac{1}{2}''$ high; inverted.

9. 25005″ in front; diameter, 5″.

## IV. (Pages 45, 46.)

1. $\sqrt{2}$. 2. $\frac{\sqrt{3}}{3}$. 3. Almost 90°. 4. $\frac{\mu_2}{\mu_1}$.

5. $\frac{2}{3}$. 6. 9′ 4″. 7. 14·34″.

## V. (Pages 54, 55.)

**3.** 1·83 ; diamond ; $\sin x = \frac{1}{1\cdot 83} = \cdot 546$. **5.** ·56 **6.** 73.

**7.** 30′. **8.** 36′. **9.** 1·434. **10.** $\frac{4}{3}$. **11.** 12·5″.

## VI. (Page 65.)

**1.** At infinity, *i.e.* the rays emerge parallel to one another.

**2.** 10″ in front of lens. **3.** 24″ behind the lens.

**4.** $10\frac{2}{7}''$ in front of lens. **5.** $5\frac{1}{15}''$ in front of lens.

**6.** $19\frac{1}{5}''$ behind the lens. **7.** 90″ in front of lens.

**8.** 18″ in front of lens.

## VII. (Pages 67, 68.)

**1.** $f = 6\cdot 16''$. **2.** (*a*) $f = r$; (*b*) $f = \frac{3r}{2}$. **3.** $f = -\frac{3r}{2}$.

**4.** (*a*) $f = -2r$; (*b*) $f = -3r$. **5.** (*a*) $f = 2r$; (*b*) $f = 3r$.

**6.** $f = -11\cdot 52''$. **7.** The light will converge to a point 24″ behind the lens.

## VIII. (Page 71.)

**1.** 7·2″ in front of lens. **2.** 20″ in front of lens.

**3.** Divergent; 30″ in front of lens. **4.** $3\frac{3}{7}''$ behind the lens.

**6.** 24″ in front.

## IX. (Pages 80, 81.)

**1.** Yes. **2.** No. **4.** Yes.

**7.** 24″ in front of lens; height of image is 3″; the image is enlarged and virtual.

**8.** Image is smaller and virtual, and is 6″ in front of lens.

**9.** $8\frac{1}{3}''$ in front of the concave lens; image is not inverted.

**10.** $5\frac{1}{7}$ cms. behind *B*.

**11.** Lens is convex, and of focal length, − 12″; the image is real and inverted.

**12.** 6·46″ behind *B*. **13.** $6\frac{3}{8}''$ behind the second lens; yes.